

# MHD FLOW OF NANOFUIDS (Cu and TiO<sub>2</sub>) IN THE PRESENCE OF POROUS MEDIA, RADIATION AND HEAT GENERATION THROUGH A VERTICAL CHANNEL

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*Abstract* - The present study investigates the effects of heat generation and radiation in unsteady MHD mixed convective flow of nanofluids along a upstanding channel in the presence of porous medium. In this study we investigate Water based – Cu and TiO<sub>2</sub>, here Water is taken as convective base fluid, Cu and TiO<sub>2</sub> particles are incorporated in this base fluid. Keeping in mind the congregating and random motion of nanoparticles, “Koo and Kleinstreuer model” is used. The main characteristics of the nanofluids have finer thermo physical properties such as high thermal conductivity, minimal blocking in flow passage, long term solidity and uniformity. The governing boundary layer equations are formulated and reduced to a set of ordinary differential equations using perturbation technique. Solutions are obtained for motion and temperature, discussed and pertinent results are shown through graphs.

*Key words*- Radiation, Heat generation, Mixed convection, Nanofluids, Brownian motion.

## I. INTRODUCTION

Nanofluids are a different type of heat transfer fluids which contain a base fluid and nanoparticles. This term nanofluid is defined as a mixture of nanoparticles of Solid metallic materials, such as Ag, Cu and Fe, and non-metallic materials, such as Al, CuO and carbon nanotubes attached in a base fluid like Water, toluene, C<sub>2</sub>H<sub>6</sub>O<sub>2</sub> or oil. The term nanofluid was introduced by Liao (1992) in 1995, he found that the thermal conductivity will be increased if some amount of nanoparticles is added to the base fluid. After that many researchers tried to investigate for heat transfer improvement in different thermal applications of nanofluids. Today, nanofluids have wide range of applications in Biomedical (drug delivery), Cancer Therapeutics, Antibacterial Agent (textile industry, water disinfection, medicine, and food packaging), Detergency, Solar Industry, Industrial Cooling Applications, Smart Fluid, Nuclear Reactors, Microchips Electronic Industry. Steady two-dimensional Steady flow through a very porous medium in the presence of free convection and radiation bounded by a standing infinite porous plate Raptis [21]. Sahoo et al. [22] studied the nano-fluid stability, zeta ( $\zeta$ ) potential, and absorbency were measured under different pH values and PVP surfactant concentrations also thermal conductivity enhancement was measured based on different volume fraction of CuO nanoparticles and temperature. Xuan et al. [29] examined CuO nanofluid with laminar forced convection. Das [7] for evaluated temperature with increment of four nano fluids with H<sub>2</sub>O as base fluid and

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particles of Alumina or Copper Oxide as suspension material. Xuan et al. [29] described their result by studied the effect of random motion, measurements and congregating of nanoparticles.

Koo and Kleinstreuer [14] generate a model to study the result of random motion of nanoparticles. The effect of the nanoparticle/base-fluid relative velocity more significantly with compare to dispersion models Buongiorno [6]. MHD oscillatory fluid flow between parallel plates with porosity Makinde, O D and Mhone P Y [16]. Under turbulent flow conditions application of CuO water nanofluid in Automotive diesel engine radiator is experimented by Navid Bozorgan et al. [17]. Temperature distribution in upstanding plates partway filled with porous medium of nanofluids Hajipour M. and Dehkordi , A. M. [12]. Flow of a fluid with natural convection flow which includes heat generation / absorption and ramped wall temperature Jha, B. K., et al [13]. In a lid driven square cavity filled with nanofluids Laminar mixed convection heat transfer is investigated by Z. Said, H.A. Mohammed, R. Saidur [18]. MHD nanofluid flow of stretchable sheet as well as moving permeable flat plate with various effects Hail E. and Shankar B. [10,11]. MHD flow and heat transfer of Ferrofluid along a stretching cylinder in presence flux of heat Qasim, M., et al. [20]. Heat Transfer of Nano fluid flow past a moving flat plate Turkyilmazoglu, M. [25]. Flow of nano fluids with free convection including boost wall temperature Asma, K., et al [4]. Inside a horizontal tube Heat transfer and pressure drop characteristics of water based CuO nanofluid investigated by Bayram et al. [5]. MHD nanofluid flow and temperature distribution past a moving standing plate Das, S., Jana, R. N. [8]. Basics of Nanofluids and applications Uddin M. J., Al Kalbani K. S., Rahman I M. M., Alam M. S., Al-Salti N. and Eltayeb I. A. [26]. Li et al. [27] examined for Thermal conductivity enhancement dependent pH and chemical surfactant for Cu-water nanofluids Radiation and heat generation effects in Magnetohydrodynamic mixed convection flow of nanofluids discussed by Aaiza , Khan & , Shafie [1]. Nano fluid flow with temperature and concentration profile including chemical reaction effect Abbas [2]. In rectangular microchannels heat sink using nanofluids flow for cooling of electronics device under uniform heat flux condition investigated by Arvind et al. [3]. MHD flow of nanofluid through upstanding plates with heat creation Prakash, D. , Suriyakumar, P. [19]. MHD flow of nanofluids with mixed convection including effect of radiation and heat generation Gul, A., Khan, I. and Shafie, S. [9]. Study of temperature distribution in Casson Nanofluid in presence of Uniform Heat Source Sink and Convective Condition Kumar, K. G., Gireesha, B. J., Krishnamurthy, M. R. and Prasannakumara, B. C. [15]. Nanofluid flow and heat transfer with immunization through enlarging and shrinking porous parallel plates Olawale, O. J. [18]. Inside a absorbent enclosure via non-equipoise model which includes flow of H<sub>2</sub>O based nanofluid flow Sheikholeslami, M. and Shehzad, S. A. [23]. Hayat and Nadeem [24] examined the impact of hybrid nano fluid in thermal radiation, heat generation and chemical reaction over stretching sheet in the presence of rotation. Nanofluid flow through parallel plates in presence of slip Xu H., Cui J. [28]. Now a days scientists all over the world are attempting to reach at an agreement by publishing their theoretical and experimental investigations so that the use of nanofluids will be effective as it was predicted before.

## II. FORMULATION OF THE PROBLEM

In present study, behavior of water based nanofluid which is containing Copper and Titanium dioxide nano particles is investigated. In the direction of the flow oscillatory type of pressure gradient is applied. Effect of heat generation parameter and radiation parameter is also taken into account. The width of vertical channel is taken as  $d$  and the channel is assumed to be filled with porous medium. Due to polarization, external electric field and electric field is assumed null. Induced magnetic field is also neglected, by taking magnetic Reynolds number very small. One partition of the channel is maintained at sustained temperature  $T_w$  while other boundary has time dependent temperature  $T_0 + \varepsilon (T_0 - T_w) e^{i\omega t}$  where  $\varepsilon (\ll 1)$  is a positive quantity. In the normal direction of the flow  $y$ - axis is taken while along the channel  $x$ -axis is taken. A transversal magnetic field of constant strength  $B_0$  is applied in the direction of the flow, due to this fluid has low electrical conductivity. So now, when electrical conductivity which the fluid have is very small and the produced electromagnetic force is also small. Then, assuming Boussinesq fluid model, the equations governing fluid motion are given as:

$$\rho_{nf} \frac{\partial u}{\partial t} = -\frac{\partial p}{\partial x} - \sigma_{nf}^2 \frac{\partial^2 u}{\partial y^2} - \frac{\mu_{nf}}{K_0} u + B_0 u + (\rho\beta)_{nf} g (T - T_0) + \mu_{nf} \frac{\partial^2 T}{\partial y^2} - \frac{\partial q}{\partial y} \quad (1)$$

$$\left(\rho c_p\right)_{nf} \frac{\partial T}{\partial t} = k_{nf} \frac{\partial^2 T}{\partial y^2} + Q_0 (T - T_0) - \frac{\partial q}{\partial y} \quad (2)$$

along with boundary conditions

$$u(0,t) = 0, \quad T(0,t) = T_0 + \varepsilon (T_0 - T_w) e^{i\omega t} \quad (3)$$

$$u(d,t) = 0, \quad T(d,t) = T_w \quad (4)$$

where the subscript  $nf$  is used for nanofluids

$u = u(y,t)$ in $x$ - direction it shows velocity of fluid	$(\rho\beta)_{nf}$ --the thermal expansion of the nanofluid
$T = T(y,t)$ is the nanofluid temperature	$g$ -- the acceleration generates through gravity
$(c_p)_{nf}$ -- the nanofluid's specific heat of at constant pressure	$\rho_{nf}$ -- the nanofluid's density
$\mu_{nf}$ -- the dynamic viscosity of the nanofluid	$k_{nf}$ -- the thermal conductivity of the nanofluid
$\sigma_{nf}$ -- the electrical conductivity of the nanofluid	$q$ -- the radiative heat flux in $x$ -direction

Now using the reference [4, 8, 20, 25] the expression for  $\rho_{nf}$ ,  $(\rho\beta)_{nf}$ ,  $(\rho c_p)_{nf}$ ,  $\sigma_{nf}$ ,  $k_{nf}$  and  $\sigma$  are derived and given by

$\rho_{nf} = \phi \rho_s + (1 - \phi) \rho_f +$	$\sigma_{nf} = \sigma_f \left[ 1 + \frac{3(\sigma - 1)\phi}{(\sigma + 2) - (\sigma - 1)\phi} \right]$
$(\rho\beta)_{nf} = \phi(\rho\beta)_s + (1 - \phi)(\rho\beta)_f$	$k_{nf} = \alpha_{nf} (\rho c_p)_{nf}$
$(\rho c_p)_{nf} = \phi(\rho c_p)_s + (1 - \phi)(\rho c_p)_f +$	$\sigma = \frac{\sigma_s}{\sigma_f}$

(5)

where subscript  $f$  is used for base fluid, subscript  $s$  is used for solid nanoparticles,  $\phi$  is the volume fraction of the nanoparticles and heat capacitance is shown by total term  $\rho c_p$ .

Some substantial properties of nanoparticles and base fluid are given in table 1 [4, 8, 20, 25]. These values will be helpful in numeric calculation of this study.

Table-1 : Properties of basefluids and nanoparticles

Model	$c_p$	$\rho$	$k$	$\beta \cdot 10^5$	$\sigma$
Water ( $H_2O$ )	4179 $\text{kg}^{-1}\text{K}^{-1}$	997.1 $\text{kgm}^{-3}$	0.613 $\text{Wm}^{-1}\text{K}^{-1}$	21 $\text{K}^{-1}$	$5.5 \cdot 10^{-6} \text{Sm}^{-1}$
Copper (Cu)	385 $\text{kg}^{-1}\text{K}^{-1}$	8933 $\text{kgm}^{-3}$	401 $\text{Wm}^{-1}\text{K}^{-1}$	1.67 $\text{K}^{-1}$	$59.6 \cdot 10^6 \text{Sm}^{-1}$
Titanium dioxide ( $\text{TiO}_2$ )	686.2 $\text{kg}^{-1}\text{K}^{-1}$	4250 $\text{kgm}^{-3}$	8.9528 $\text{Wm}^{-1}\text{K}^{-1}$	0.9 $\text{K}^{-1}$	$2.6 \cdot 10^6 \text{Sm}^{-1}$

The generalized model of Koo and Kleinstreuer is used for effective thermal conductivity and viscosity of nanofluid:

$$k_{nanofluid} = k_{static} + k_{Brownian}$$

where

$$k_{static} = k_f \left[ \frac{(k_s + 2k_f) - 2\phi(k_f - k_s)}{(k_s + 2k_f) + \phi(k_f - k_s)} \right] \text{ and } k_{Brownian} = 5 \cdot 10^4 \beta \phi \rho_f c_{pf} \sqrt{\frac{k_b T}{\rho_s d_p}} f(T, \phi, etc.) \quad (6)$$

$$\text{and } \mu_{nanofluid} = \mu_{static} + \mu_{Brownian}$$

where

$$\mu_{static} = \frac{\mu_f}{(1 - \phi)^{2.5}} \text{ and } \mu_{Brownian} = 5 \cdot 10^4 \beta \phi \rho_f \sqrt{\frac{k_b T}{\rho_s d_p}} f(T, \phi, etc.) \quad (7)$$

$k_b$  = The Constant of Boltzman =  $1.3807 \cdot 10^{-23}$  J/K

T = Nanofluids's Temperature  $300\text{K} > T > 325\text{K}$ .

$d_p$  = Solid particles Diameter

$\beta$  = Denotes the fraction of the liquid volume, which travels with a particle and includes particle motion

$f(t, \phi, etc.)$  = It depends on particle interactions. The values of  $\beta$  and  $f(t, \phi, etc.)$  are mentioned in table [2] and [3], using the generalized Koo and Kleinstreuer model

Particle	$\beta$	Note	$f(t, \phi, etc.)$
Cu and TiO <sub>2</sub>	$0.0137(100\phi)^{-0.8229}$	$\Phi < 1\%$	1

The heat flux radiation is given by:

$$-\frac{\partial q}{\partial y} = 4\alpha^2(T - T_0) \tag{8}$$

and pressure gradient is given by

$$-\frac{\partial p}{\partial x} = \lambda \exp(i\omega t) \tag{9}$$

where  $\alpha_0$  the mean radiation absorption coefficient,  $\lambda$  constant,  $\omega_1$  the oscillation frequency.

III. MATHEMATICAL ANALYSIS

Introducing the following undimensionl variables:

$$x^* = \frac{x}{d}, y^* = \frac{y}{d}, u^* = \frac{u}{U_0}, t^* = \frac{tU_0}{d}, P^* = \frac{d}{\mu_f U} p, T^* = \frac{T-T_0}{T_w-T_0}, \omega^* = \frac{d\omega_1}{U_0} \tag{10}$$

The system of equations (1) to (4) reduces to:

$$a_0 \frac{du}{dt} = \lambda \varepsilon \exp(i\omega t) + \frac{d^2u}{dy^2} - a_3 u + a_1 T \tag{11}$$

$$u(0, t) = 0, u(1, t) = 0, t > 0 \tag{12}$$

$$b^2 \frac{dT}{dt} = \frac{d^2T}{dy^2} + b^2 T \tag{13}$$

$$T(0, t) = \varepsilon e^{i\omega t}, T(1, t) = 1, t > 0 \tag{14}$$

where

$$a_0 = \phi Re_1, \phi_1 = (1 - \phi) + \phi \frac{\rho_s}{\rho_f}, \phi_2 = \frac{1}{(1 - \phi)^{2.5}} + 5.10^4 \beta \phi \frac{\rho_f}{\mu_f} \sqrt{\frac{K_b T}{\rho_s d_p}} f(t, \phi, etc.),$$

$$\phi_3 = (1 - \phi) + \phi \frac{(\rho\beta)_s}{(\rho\beta)_f}, \phi_5 = 1 + \frac{3(\sigma - 1)\phi}{(\sigma + 2) - (\sigma - 1)\phi}, Re = \frac{U_0 d}{\nu},$$

$$M^2 = \frac{\sigma_f}{\mu_f} B^2 d^2, m_0^2 = \phi_5 M^2, \frac{1}{K} = \frac{d^2}{K}, Gr = \frac{\beta_f d^2}{\nu_f U_0} g(T_w - T_0),$$

$$a_1 = \phi \frac{(\rho\beta)_f}{\mu_f} g \frac{d^2}{U_0} (T_w - T_0) = \phi Gr, a_3 = m^2 + \frac{\phi_2}{K}, \phi_4 = (1 - \phi) + \phi \frac{(\rho C_p)_s}{(\rho C_p)_f}$$

$$Pe = (\rho C_p) \frac{U_0}{d}, \lambda = [\frac{(K_s + 2K_f) - 2\phi(K_f - K_s)}{(K_s + 2K_f) + \phi(K_f - K_s)} + 5.10^4 \beta \phi \frac{\rho_f}{\mu_f} (C_p) \sqrt{\frac{K_b T}{\rho_s d_p}} f(t, \phi, etc.)]$$

$$Q = \frac{d^2 Q_0}{K_f}, N^2 = \frac{4\alpha^2 d^2}{K_f}, b^2 = \frac{\phi Pe}{K_f}, b^2 = \frac{Q + N^2}{K_f}, m^2 = \frac{a_3}{K_f}, a = \frac{a_1}{K_f}, \lambda = \frac{\lambda}{K_f}$$

$$K_f \quad 0 \quad \lambda_n \quad 1 \quad \lambda_n \quad \begin{matrix} 1 & 2 \\ \phi & \phi_2 \end{matrix} \quad \begin{matrix} 1 \\ \phi_2 \end{matrix}$$

Re Reynolds Number, M magnetic parameter, Gr thermal Grashof Number, Pe Peclet Number, Q heat generation Parameter and N Radiation Parameter.

#### IV. METHOD OF SOLUTION

Equations (10) to (13) show that their solutions are not easily tracible. Therefore, perturbation method is a global approach to find the solution of such differential equations.

Evidently, the parameter  $\varepsilon$  is assumed to be small such that  $0 < \varepsilon < 1$ . In order to solve equations (10) to (13) assuming

$$u(y,t) = u_0(y) + \varepsilon u_1(y)e^{i\omega t}, \quad (15)$$

$$T(y,t) = T_0(y) + \varepsilon T_1(y)e^{i\omega t}. \quad (16)$$

where  $u_0$ ,  $T_0$  and  $u_1$ ,  $T_1$  denote steady and unsteady parts of velocity and temperature distribution respectively.

Through straight forward calculations, the solutions of the equations (11) to (14) with the help of equations (15) and (16) are known and given by

$$T(y,t) = \frac{\sin b_1 y}{\sin b_1} + \varepsilon \frac{\sin(m_3 - m_3 y)}{\sin m_3} e^{i\omega t} \quad (17)$$

$$u(y,t) = -\frac{a_2}{b_1 + m_1} \frac{\sinh m_1 y}{\sinh m_1} + \frac{a_2}{b_1 + m_1} \frac{\sin b_1 y}{\sin b_1} + \varepsilon \left( \frac{J_1 e^{m_1 y} + J_2 e^{-m_1 y} + \frac{a_2}{m_3 + m_2} \left\{ \frac{\sin(m_3 - m_3 y)}{\sin m_3} \right\} + \frac{\lambda_1}{m_2}}{m_3 + m_2} \right) e^{i\omega t} \quad (18)$$

where

$$m_2 = \sqrt{\frac{a_3 + a_0 i\omega}{\phi_2}}, \quad m_3 = \sqrt{b_1^2 - b_0^2 i\omega}, \quad J_1 = \frac{a}{m_2^2} + m_2^2 \left[ \frac{e^{-m_2}}{e^{m_2} - e^{-m_2}} \right] \frac{\lambda}{m_2^2} \left[ \frac{1 - e^{m_2}}{e^{m_2} - e^{-m_2}} + 1 \right],$$

$$J_2 = -\frac{a}{m_3^2} + m_3^2 \left[ \frac{e^{m_2}}{e^{m_2} - e^{-m_2}} \right] + \frac{\lambda_1}{m_2^2} \left[ \frac{1 - e^{m_2}}{e^{m_2} - e^{-m_2}} \right]$$

Rate of change of velocity and heat transfer are calculated from equations (17) and (18)

$$(Nu)_{y=0} = \frac{b_1}{\sin b_1} - \varepsilon m_3 \frac{\cos m_3}{\sin m_3} e^{i\omega t} \quad (19)$$

$$(\tau)_{y=0} = -\frac{a_2 m_1}{(b_1^2 + m_1^2) \sinh m_1} + \frac{a_2 b_1}{(b_1^2 + m_1^2) \sin b_1} + \varepsilon \left( \frac{J_1 m_1 - J_2 m_1 - \frac{a_2 m_3}{m_3^2 + m_2^2} \frac{\cos m_3}{\sin m_3}}{m_3^2 + m_2^2} \right) e^{i\omega t} \quad (20)$$

V. RESULTS AND DISCUSSION:

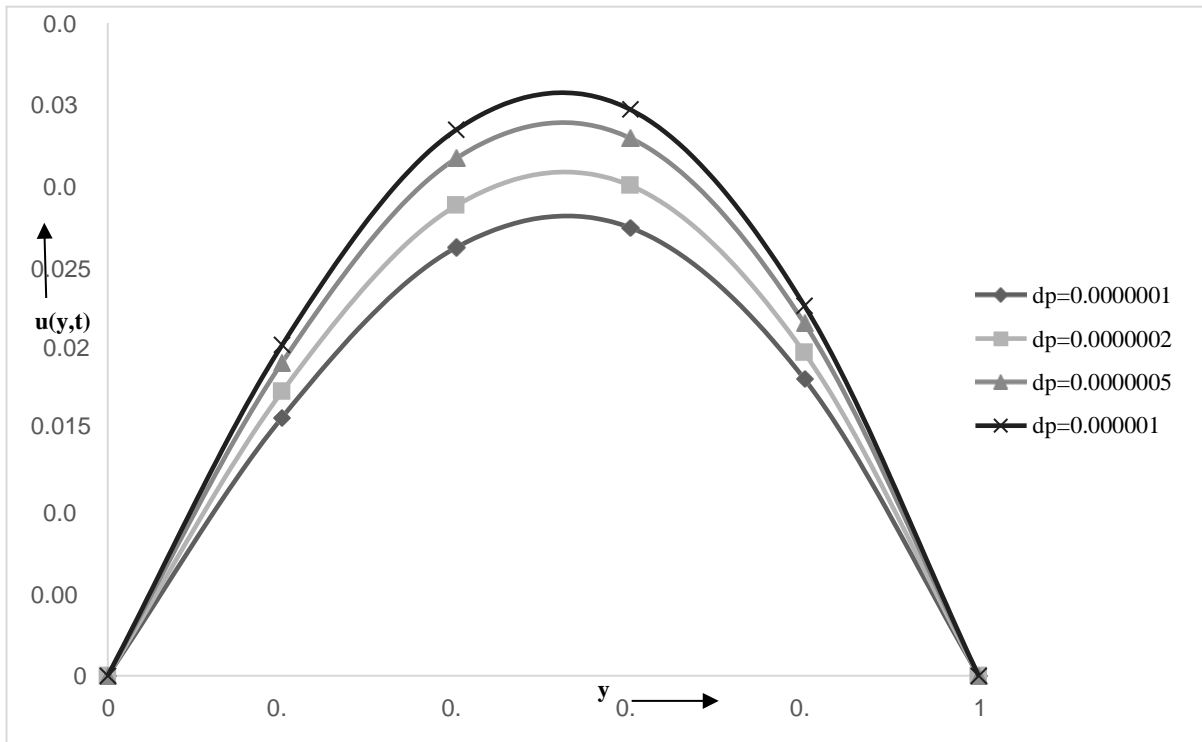


Fig. 1: Velocity profile for varying size of Cu particles in water based nanofluid when  $Gr = 0.2$ ,  $Re = 2$ ,  $Pe = 2$ ,  $M = 2$ ,  $\omega = 0.2$ ,  $t = 0.1$ ,  $\varepsilon = 0.01$ ,  $Q = 5$ ,  $\lambda = 1$ ,  $N = 2$ ,  $k_0 = 2$ ,  $\phi = 0.04$ .

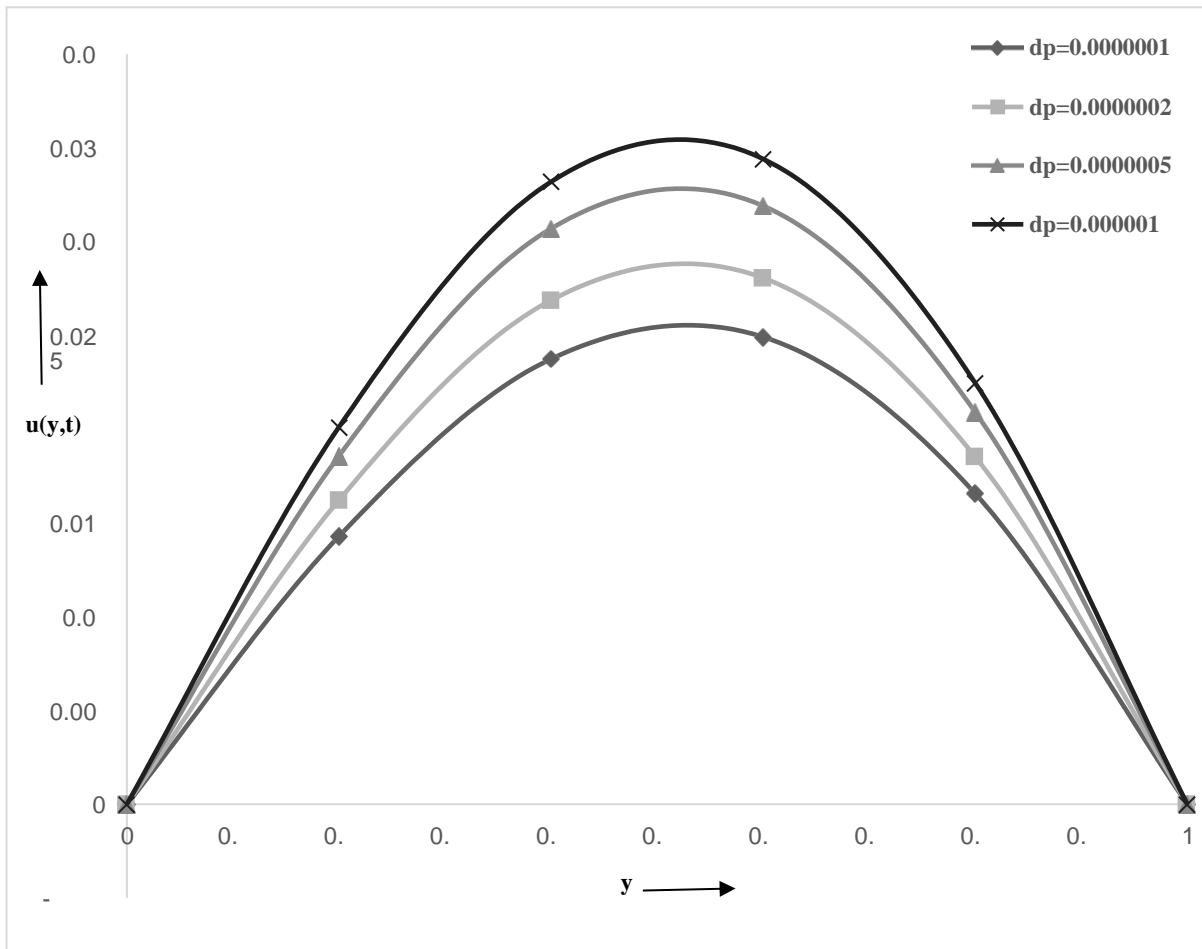
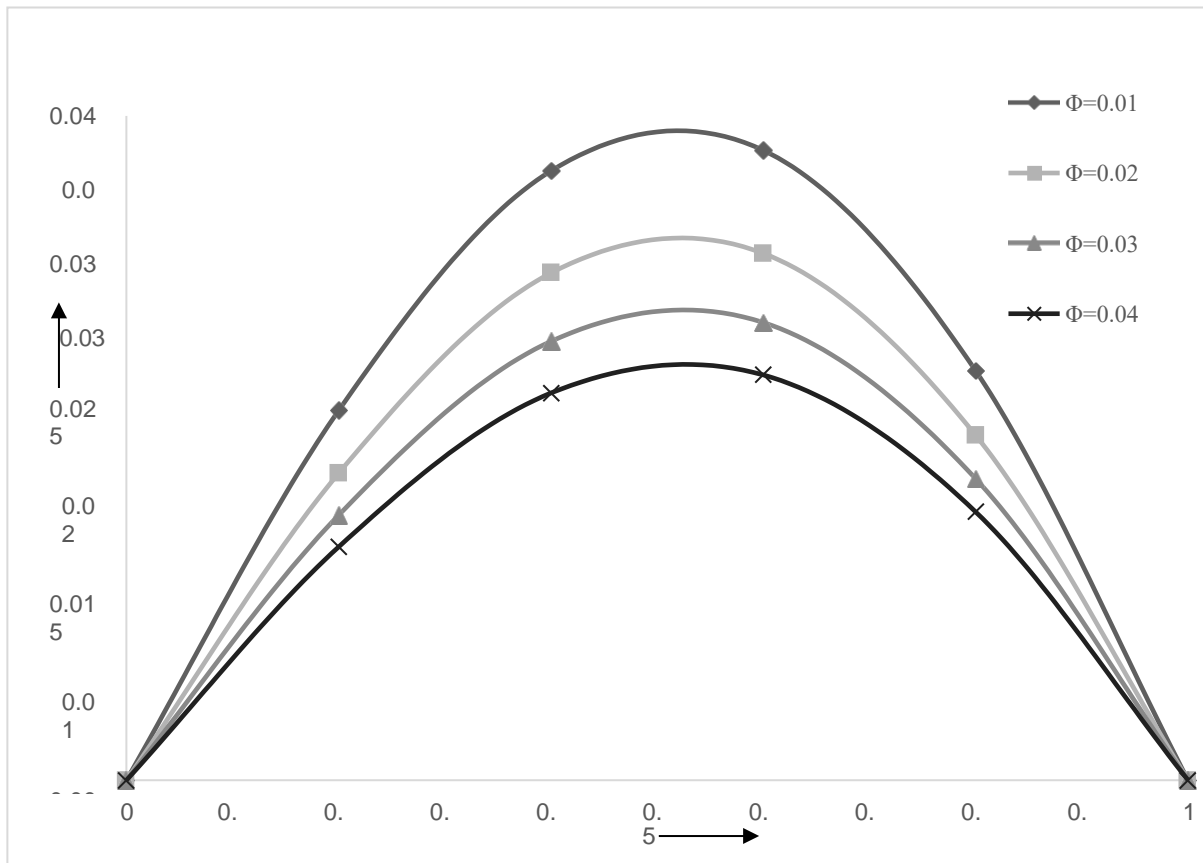
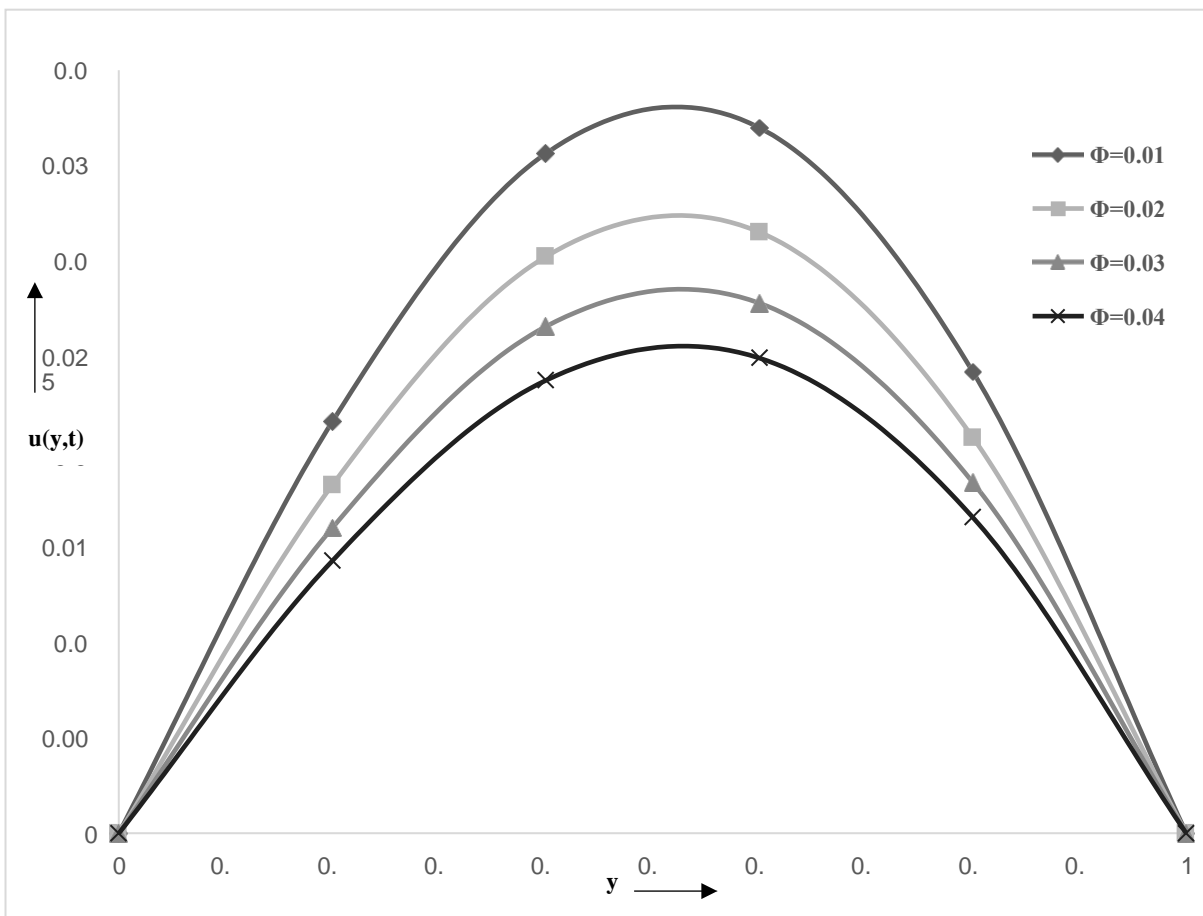


Fig. 2: Velocity profile for varying size of  $TiO_2$  particles in water based nanofluid when  $Gr = 0.2$ ,  $Re = 2$ ,  $Pe = 2$ ,  $M = 2$ ,  $\omega = 0.2$ ,  $t = 0.1$ ,  $\varepsilon = 0.01$ ,  $Q = 5$ ,  $\lambda = 1$ ,  $N = 2$ ,  $k_0 = 2$ ,  $\phi = 0.04$ .

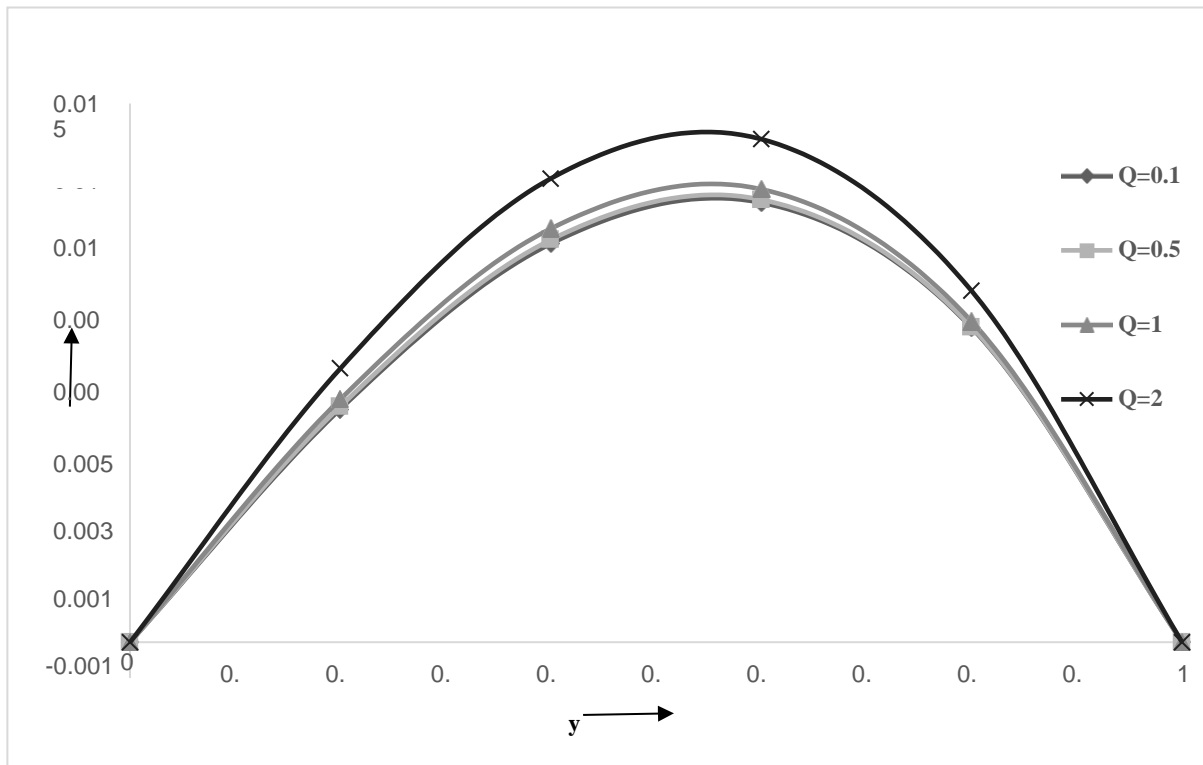




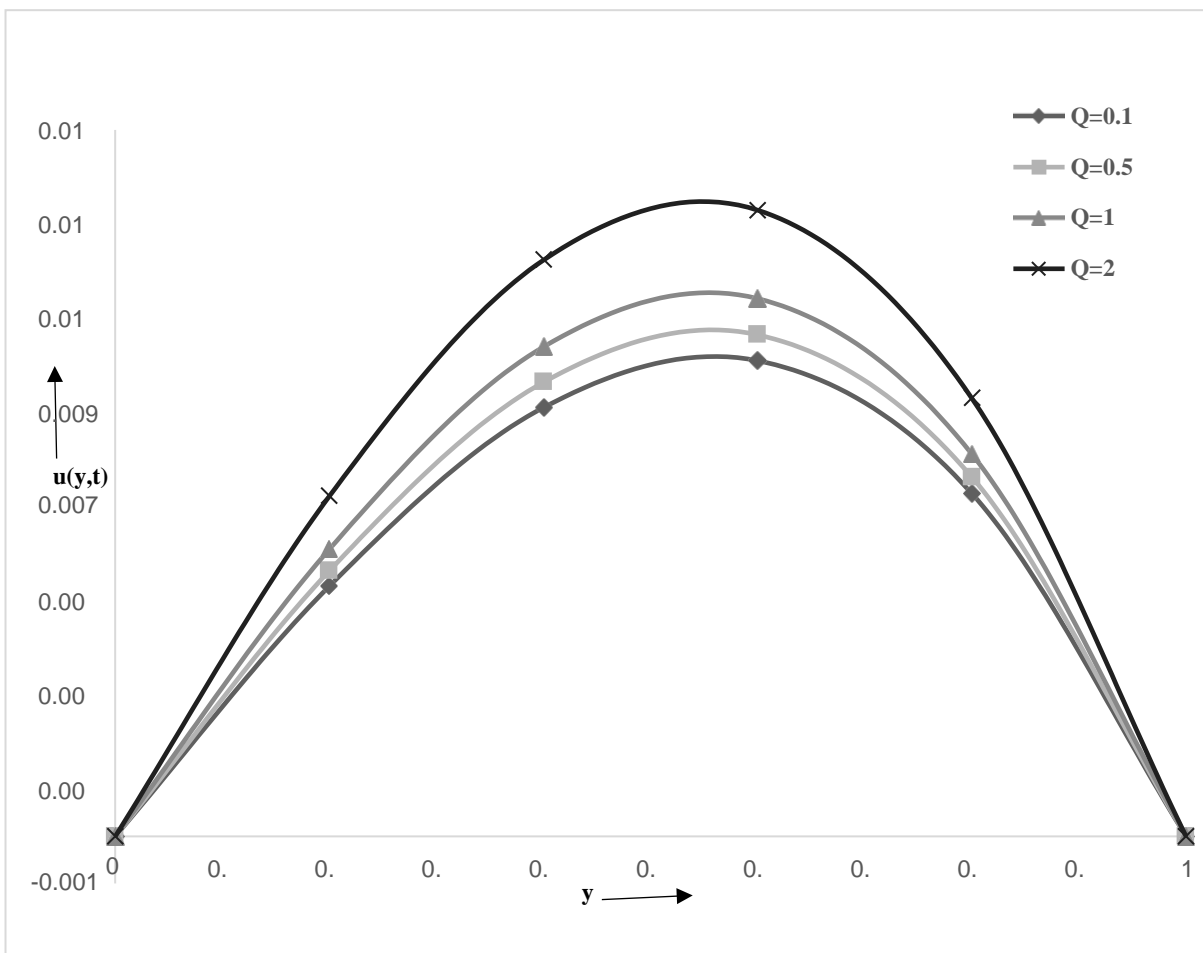
'Fig. 3' : Velocity profile for varying  $\phi$  of Cu particles in water based nanofluid when  $Gr = 0.2$ ,  $Re = 2$ ,  $Pe = 2$ ,  $M = 2$ ,  $\omega = 0.2$ ,  $t = 0.1$ ,  $\epsilon = 0.01$ ,  $Q = 5$ ,  $\lambda = 1$ ,  $N = 2$ ,  $k_0 = 2$ ,  $d = 100$  nm.



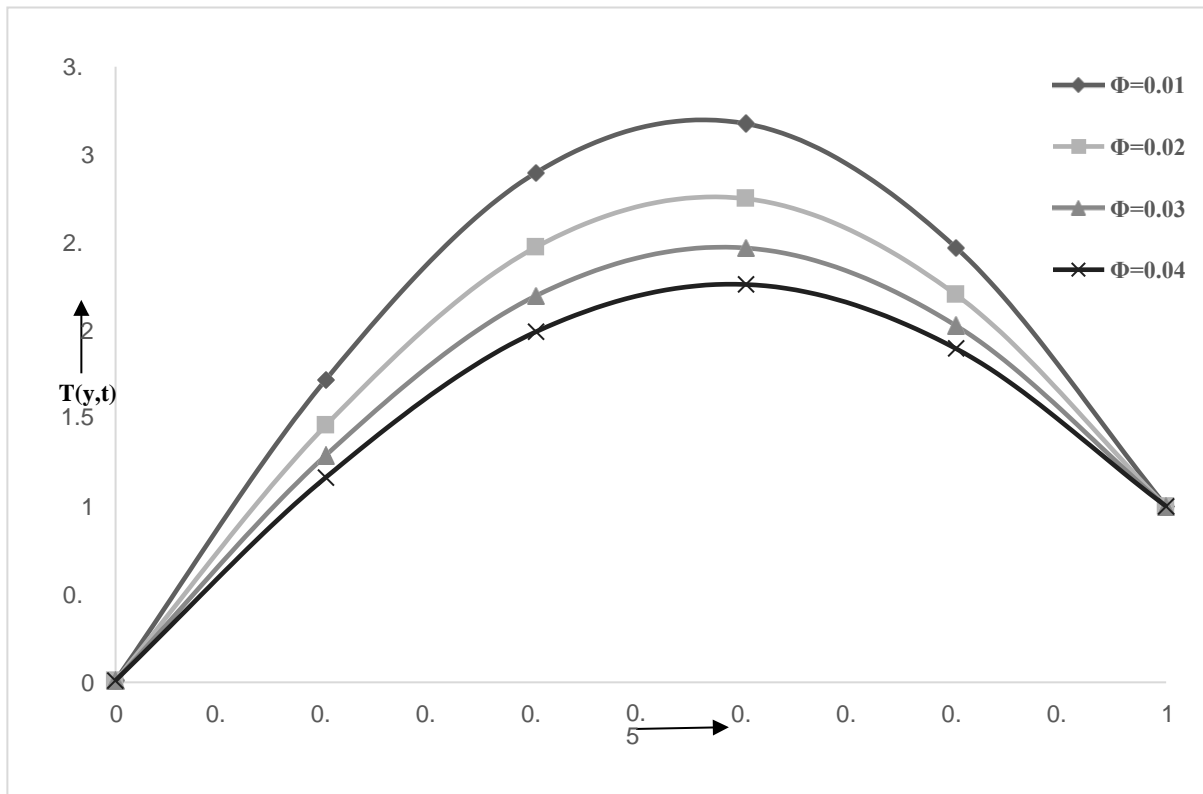
'Fig. 4' : Velocity profile for varying  $\phi$  of  $TiO_2$  particles in water based nanofluid when  $Gr = 0.2$ ,  $Re = 2$ ,  $Pe = 2$ ,  $M = 2$ ,  $\omega = 0.2$ ,  $t = 0.1$ ,  $\epsilon = 0.01$ ,  $Q = 5$ ,  $\lambda = 1$ ,  $N = 2$ ,  $k_0 = 2$ ,  $d = 100$  nm.



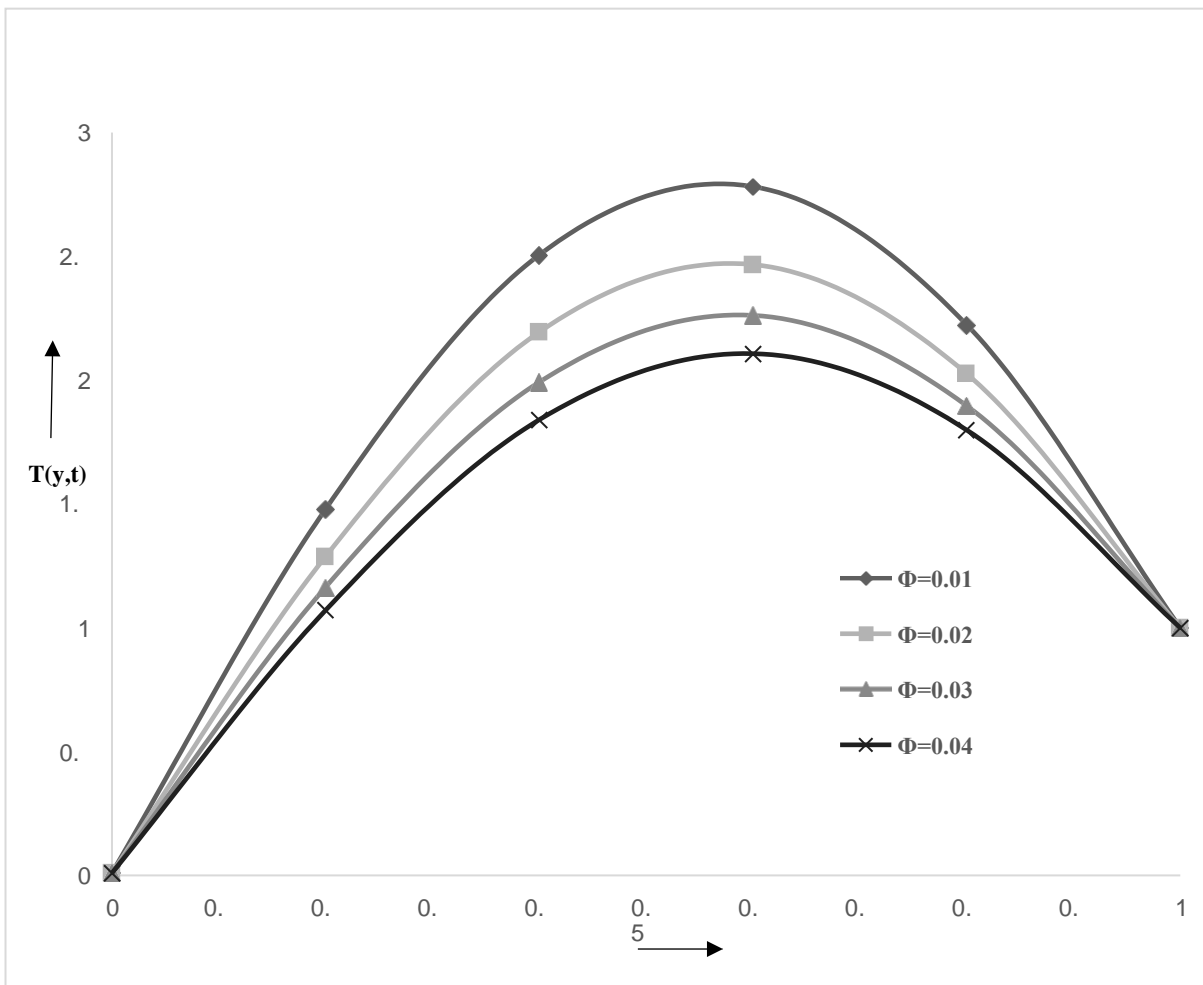
'Fig. 5' : Velocity profile for varying Q of Cu particles in water based nanofluid when  $Gr = 0.2$ ,  $Re = 2$ ,  $Pe = 2$ ,  $M = 2$ ,  $\omega = 0.2$ ,  $t = 0.1$ ,  $\epsilon = 0.01$ ,  $\phi = 0.04$ ,  $\lambda = 1$ ,  $N = 2$ ,  $k_0 = 2$ ,  $d = 100$  nm.



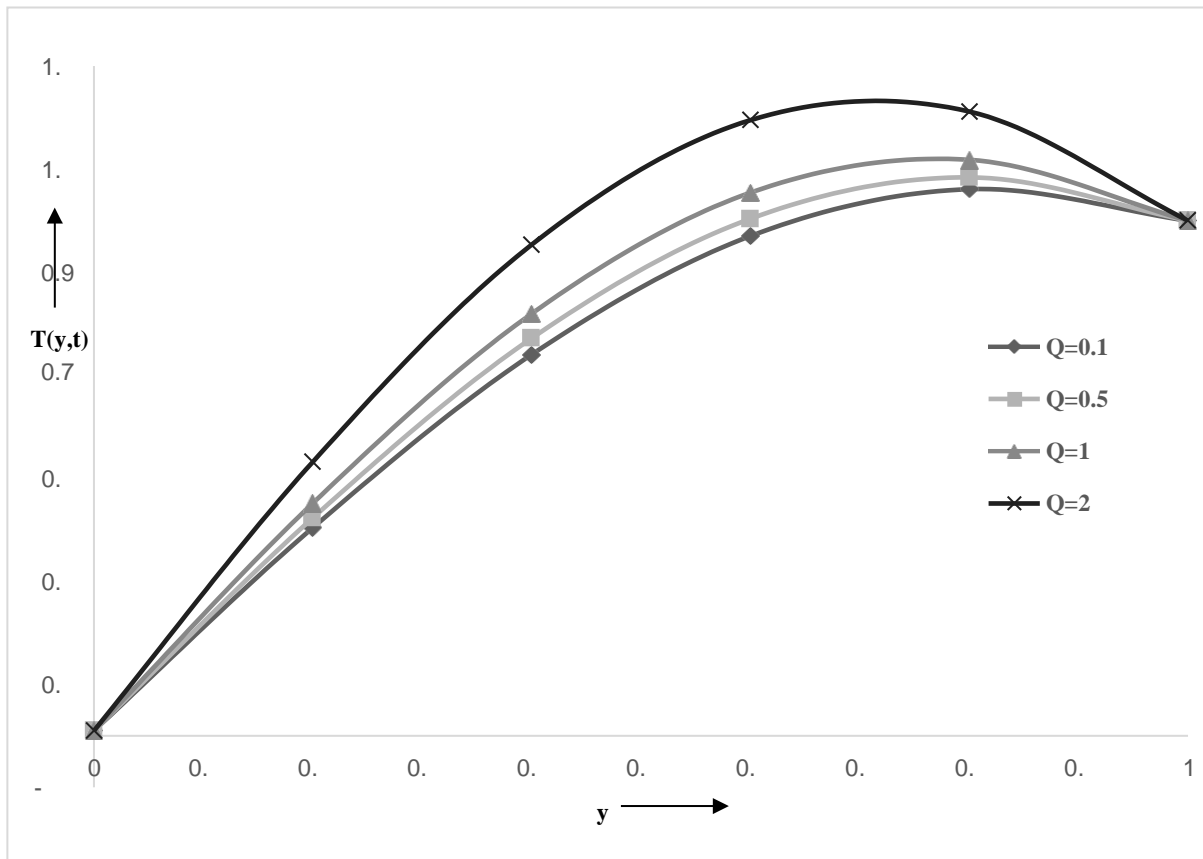
'Fig. 6' : Velocity profile for varying Q of  $TiO_2$  particles in water based nanofluid when  $Gr = 0.2$ ,  $Re = 2$ ,  $Pe = 2$ ,  $M = 2$ ,  $\omega = 0.2$ ,  $t = 0.1$ ,  $\epsilon = 0.01$ ,  $\phi = 0.04$ ,  $\lambda = 1$ ,  $N = 2$ ,  $k_0 = 2$ ,  $d = 100$  nm.



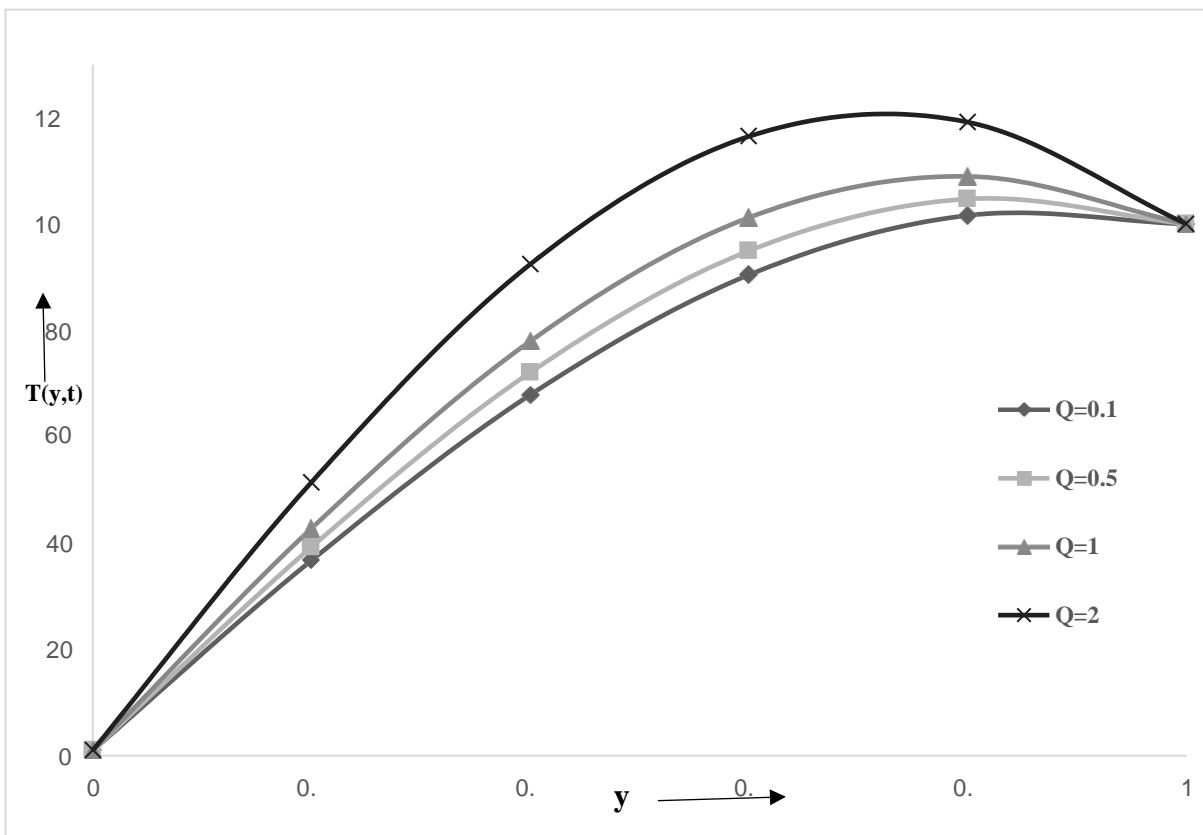
'Fig. 7' : Temperature profile for varying  $\phi$  of Cu particles in water based nanofluid when  $Gr = 0.2$ ,  $Re = 2$ ,  $Pe = 2$ ,  $M = 2$ ,  $\omega = 0.2$ ,  $t = 0.1$ ,  $\epsilon = 0.01$ ,  $\lambda = 1$ ,  $Q = 5$ ,  $N = 2$ ,  $k_0 = 2$ ,  $d = 100$  nm.



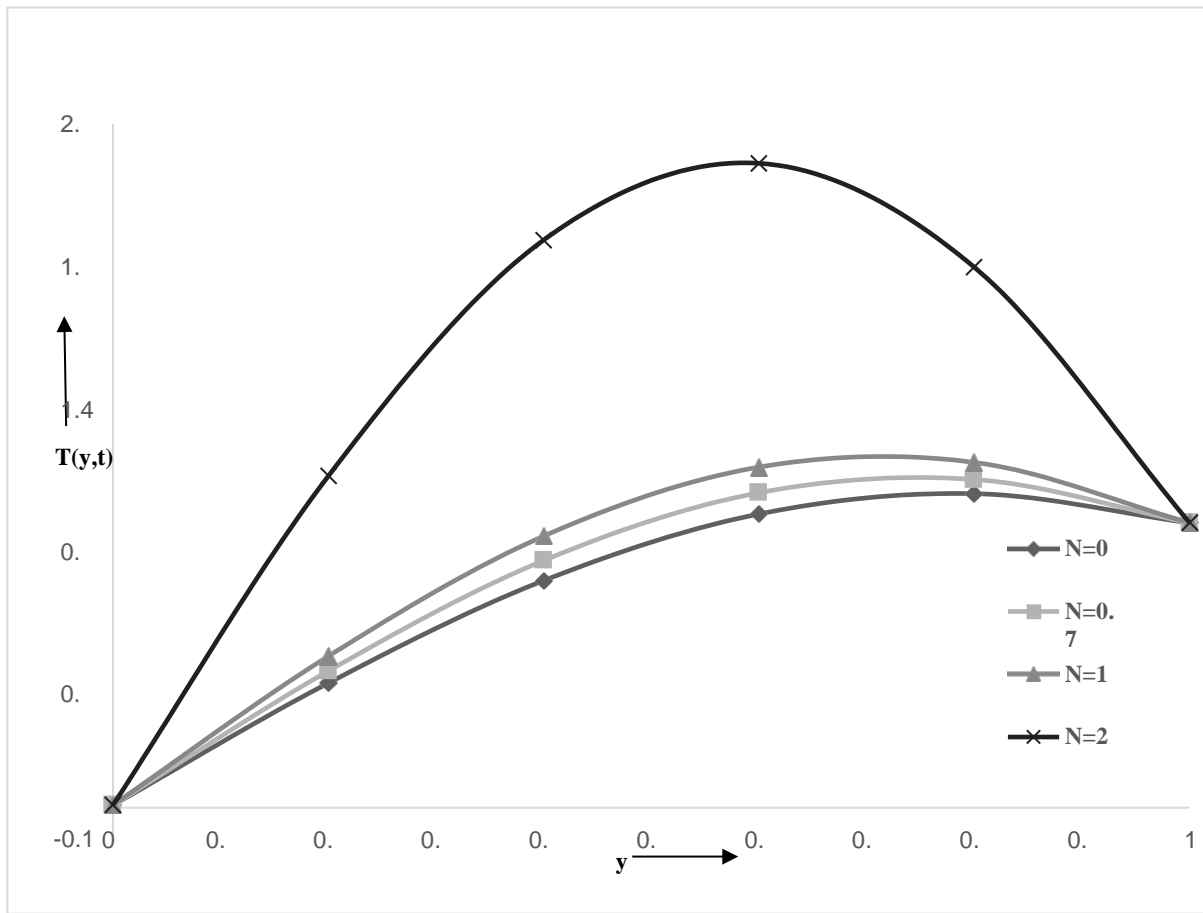
'Fig. 8' : Temperature profile for varying  $\phi$  of  $TiO_2$  particles in water based nanofluid when  $Gr = 0.2$ ,  $Re = 2$ ,  $Pe = 2$ ,  $M = 2$ ,  $\omega = 0.2$ ,  $t = 0.1$ ,  $\epsilon = 0.01$ ,  $\lambda = 1$ ,  $Q = 5$ ,  $N = 2$ ,  $k_0 = 2$ ,  $d = 100$  nm.



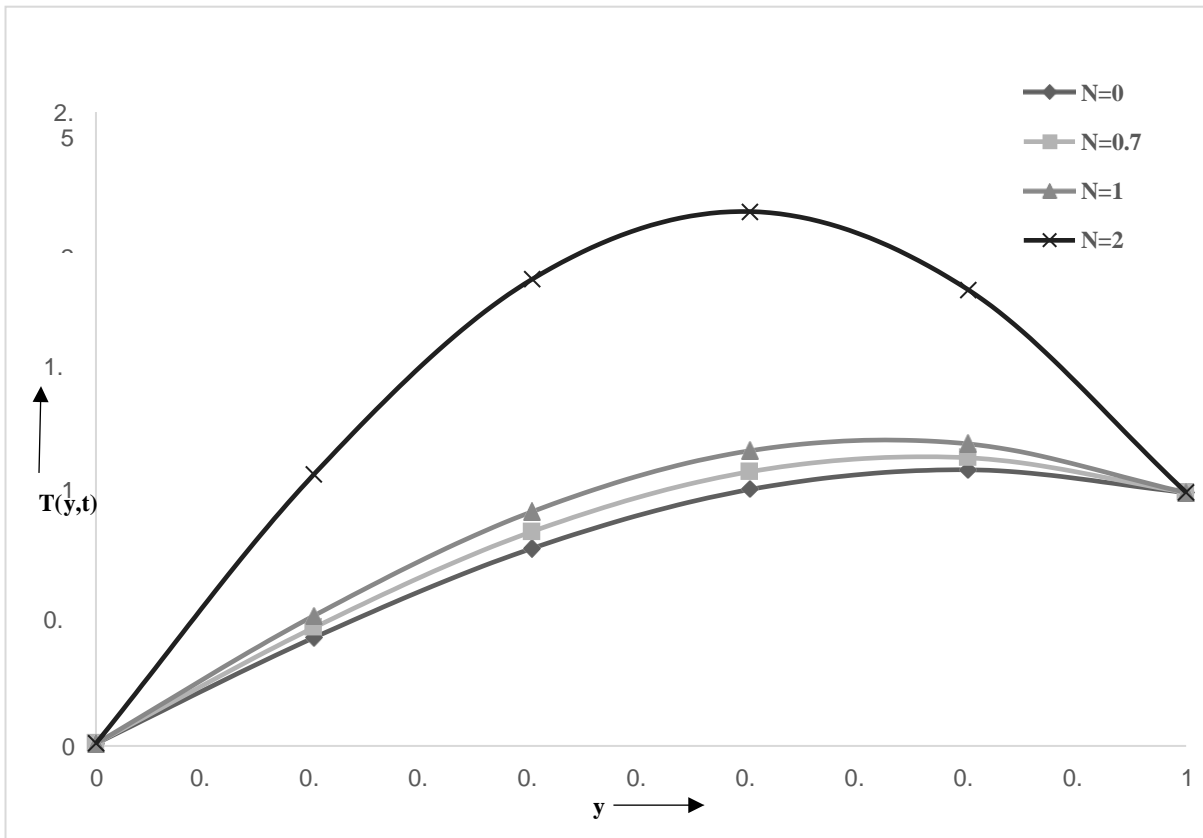
'Fig. 9' : Temperature profile for varying Q of Cu particles in water based nanofluid when  $Gr = 0.2$ ,  $Re = 2$ ,  $Pe = 2$ ,  $M = 2$ ,  $\omega = 0.2$ ,  $t = 0.1$ ,  $\epsilon = 0.01$ ,  $\lambda = 1$ ,  $\phi = 0.04$ ,  $N = 2$ ,  $k_0 = 2$ ,  $d = 100$  nm.



'Fig. 10' : Temperature profile for varying Q of  $TiO_2$  particles in water based nanofluid when  $Gr = 0.2$ ,  $Re = 2$ ,  $Pe = 2$ ,  $M = 2$ ,  $\omega = 0.2$ ,  $t = 0.1$ ,  $\epsilon = 0.01$ ,  $\lambda = 1$ ,  $\phi = 0.04$ ,  $N = 2$ ,  $k_0 = 2$ ,  $d = 100$  nm.



'Fig. 11' : Temperature profile for varying value of N for Cu particles in water based nanofluid when  $Gr = 0.2$ ,  $Re = 2$ ,  $Pe = 2$ ,  $M = 2$ ,  $\omega = 0.2$ ,  $t = 0.1$ ,  $\epsilon = 0.01$ ,  $\lambda = 1$ ,  $\phi = 0.04$ ,  $Q = 5$ ,  $k_0 = 2$ ,  $d = 100$  nm.



'Fig. 12' : Temperature profile for varying value of N for  $TiO_2$  particles in water based nanofluid when  $Gr = 0.2$ ,  $Re = 2$ ,  $Pe = 2$ ,  $M = 2$ ,  $\omega = 0.2$ ,  $t = 0.1$ ,  $\epsilon = 0.01$ ,  $\lambda = 1$ ,  $\phi = 0.04$ ,  $Q = 5$ ,  $k_0 = 2$ ,  $d = 100$  nm.

## VI. DISCUSSION

Graphical results with discussion are included in this section. Koo and Kleinstreuer model is used to derive various results for silver and alumina – water based nano fluids. Using the thermophoretic properties water, Copper and Titanium , Charts 1-12 are plotted. ‘Figures 1-6’ are plotted for motion and ‘Figures 7-12’ are plotted for temperature distribution.

‘Figure 1’ depicts motion profile of Copper nano particles of various size in H<sub>2</sub>O based nanofluids. From this figure easy to understand that fluid velocity increases due to increasing in size of Cu nano particles in water based nanofluids. The rising of motion with size of nanoparticles means that viscosity and thermal conductivity of nanofluids are decreasing. Which implies that in comparison to small size particles, large size particles has good possibility of generating clumps. ‘Figure 2’ is plotted to describe motion of TiO<sub>2</sub> in water based nanofluids. This figure shows that dissemination of Ti nanoparticles in H<sub>2</sub>O gives exactly same outcome as in study of Copper nano particles in H<sub>2</sub>O. ‘Figure 3’ shows with increase of volume fraction of nano particles in case of Copper particles the velocity of nano fluid decreases and same behavior is noted in case of TiO<sub>2</sub> as plotted in ‘Figure 4’ , also the base fluid is same in both the cases. This means when we rising volume fraction of particles viscosity of fluid also rising and the velocity of fluid reduce due to this. ‘Figure 5’ and ‘Figure 6’ are drawn to explain the variation of fluid velocity with respect to Q . Velocity profile in this case directly proportional to Q, as Q increases velocity of nano fluid is also rising. ‘Figure 7’ and ‘Figure 8’ evaluate temperature profile of nano fluid with varying volume fraction. It is obvious from the graph that temperature of nanofluids decreases with increase of volume fraction. ‘Figure 9’ and ‘Figure 10’ is drawn to explain the variation of fluid temperature with respect to Q . Temperature profile in this case proportional to Q, as Q increases temperature of nano fluid also rises. ‘Figure 11’ and ‘Figure 12’ depicts the variation of radiation parameter with respect to temperature profile. Logically, the rate of heat transfer of the fluid increases on rising radiation parameter, hence the temperature increases.

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