Penta pre-continuity and Penta semi-continuity in penta Topological Space

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Abstract.

The aim of this paper is to study properties of penta pre-open set and penta semi-open sets in penta topological spaces and also introduce penta pre-continuity and penta semi-continuity in penta topological space.

Keywords: penta semi-open sets, penta semi-continuity, penta pre-open sets, penta pre-continuity.

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1. Introduction

J .C. Kelly [2] introduced the concept of bitopolgical space. N. Levine [5] introduced the idea of semi-open sets and semi-continuity and Mashhour et. al [7] introduced the concept of pre-open sets and pre-continuity in a topological space. M. Jelic [1] generalized the idea of pre- open sets and pre-continuity in bitopological space. S.N. Maheshwari and Prasad [6] introduced semi-open sets in bitopological spaces. The tri topological space was introduced by Martin M. Kovar [4].S. Palaniammal [10] studied properties of tri-open sets in tri-topological space. D.V. Mukundan [8] introduced quad topological space.

Penta topological space was introduced by Muhammad Shahkar Khan and Gulzar Ali Khan [3]. G. Priscilla Pacifica & S. Shehnaz Fathima [9] studied basic concepts in penta topological space. In this paper we use penta-open set(p-open set) in place of $\tau_i: i \in \{1,2,3,4,5\}$ open set.

In this paper, we studied penta semi-open sets, penta pre-open sets, penta semi-continuity and penta pre-continuity and their fundamental properties in penta topological space.

2. Preliminaries

Definition 2.1[3] Let (X, τ_p) be a p-topological space. Elements of τ_i; $i \in 1, 2, 3, 4, 5$ are called τ_i-open sets and their relative complements are called τ_i-*closed sets*.

Definition 2.2[3]: Let (X, τ_p) be a p-topological space. A subset A of X is called penta-open (p-open) if $A \in \cup \tau_i$; $i \in \{1, 2, 3, 4, 5\}$ and its complement is said to be penta-closed (p-closed).

Definition 2.3[9]: Let $(X, \tau_1, \tau_2, \tau_3, \tau_4, \tau_5)$ be a penta topological space. $A \subset X$ is called semi (1, 2,3,4, 5) open if $A \subset \tau_{1,2,3,4,5}\, cl \tau_{1,2,3,4,5} \operatorname{int}(A)$.

Definition 2.4[9]: Let X, τ_1, τ_2, τ_3, τ_4, τ_5 be a penta topological space. $A \subset$ X is called pre (1, 2,3,4, 5) open if $A \subset \tau_{1,2,3,4,5} \operatorname{int} \tau_{1,2,3,4,5} cl(A)$.

3. Penta Pre-open sets in penta topological space

Definition 3.1: Let $(X, T_1, T_2, T_3, T_4, T_5)$ be a penta topological space. The intersection of all penta pre-closed sets of X containing a subset A of X is called penta pre-closure of A and is denoted by $p-P\operatorname{int}(A)$.

Definition 3.2: Let $(X, T_1, T_2, T_3, T_4, T_5)$ be a penta topological space. The intersection of all penta pre-closed sets of X containing a subset U of X is called penta pre-closure of U and is denoted by $p-Pcl(U)$.

Theorem 3.3: Let A and B be subsets of $(X, T_1, T_2, T_3, T_4, T_5)$ and $x \in X$

(i) U is penta pre-closed if and only if $A = p-Pcl(U)$

(ii) U is penta pre-open if and only if $U = p-P\operatorname{int}(U)$

(iii) $p-P\operatorname{int}(U \cup V) \supset p-P\operatorname{int} U \cup p-P\operatorname{int} V$ ·

(iv) If $U \subset V$, then $p-Pcl(U) \subset p-Pcl(V)$.

(v) $x \in p-Pcl(A)$ if and only if $A \cap U \neq \phi$ for every penta pre-open set U containing x .

Theorem 3.4: Let A be a subsets of $(X, T_1, T_2, T_3, T_4, T_5)$**,** if there exist an penta pre-open set U such that $A \subset U \subset p-cl(A)$, then A is penta pre-open.

Theorem 3.5: In a penta topological space $(X, T_1, T_2, T_3, T_4, T_5)$, the union of any two penta pre-open sets is always a penta pre-open set.

Proof: Let A and B be any two penta pre-open sets in X .

Now $A \cup B \subseteq p-\operatorname{int}\left(p-cl\left(A\right)\right) \cup p-\operatorname{int}\left(p-cl\left(B\right)\right)$

$\Rightarrow A \cup B \subseteq p-\operatorname{int}\left(p-cl\left(A \cup B\right)\right)$. Hence $A \cup B$ penta pre-open sets.

Theorem 3.6: if A is penta-open sets then A is penta pre-open set.

Proof: Let A is penta closed set.

Therefore, $A = p - cl(A)$.

Now, $A \subset p - \text{int}(A) = p - \text{int}\left(p - cl\left(A\right)\right)$.Hence A is penta pre-open set.

Theorem 3.7: Let A and B be subsets of X such that $B \subseteq A \subseteq p - \text{int}(B)$. if B is penta pre-open set then A is also penta pre-open set.

Proof: Given B is penta pre-open set. so, we have $B \subseteq p - \text{int}\left(p - cl\left(B\right)\right) \subseteq p - \text{int}\left(p - cl\left(A\right)\right)$.

Thus $p - \text{int}(B) \subseteq p - \text{int}\left(p - cl\left(A\right)\right)$.Hence A is also penta pre-open set.

4. Penta Pre-Continuity in penta topological space

Definition 4.1: A function f defined from a penta topological space $(X, T_1, T_2, T_3, T_4, T_5)$ into another penta topological space $\left(Y, W_1, W_2, W_3, W_4, W_5\right)$ is called penta pre-continuous function if $f^{-1}\left(V\right)$ is penta pre-open set in X for each penta open set V in Y.

Example 4.2: Let $X = \{a,b,c,d\}$, $T_1 = \{X, \phi, \{a\}\}$, $T_2 = \{X, \phi, \{a,d\}\}$, $T_3 = \{X, \phi, \{b,d\}\}$, $T_4 = \{X, \phi, \{a,b,d\}\}, T_5 = \{X, \phi\}$

Open sets in penta topological spaces are union of all five topologies.

Then penta open sets of $X = \{X, \phi, \{a\}, \{a,d\}, \{b,d\}, \{a,b,d\}\}$

penta pre-open set of X is denoted by $p - PO(X) = \{X, \phi, \{a\}, \{a,d\}, \{b,d\}, \{a,b,d\}\}$.

Let $Y = \{1,2,3,4\}$, $W_1 = \{Y, \phi, \{1,4\}\}$, $W_2 = \{Y, \phi, \{4\}\}$, $W_3 = \{X, \phi, \{1,2\}\}$, $W_4 = \{X, \phi, \{1,2,4\}\}$

$W_5 = \{Y, \phi\}$

penta open sets of $Y = \{Y, \phi, \{4\}, \{1,2\}, \{1,4\}, \{1,2,4\}\}$.

penta pre-open set of Y is denoted by $p - PO(Y) = \{Y, \phi, \{4\}, \{1,2\}, \{1,4\}, \{1,2,4\}\}$.

Consider the function $f : X \to Y$ is defined as

$f^{-1}\{4\} = \{a\}$, $f^{-1}\{1,2\} = \{b,d\}$, $f^{-1}\{1,4\} = \{a,d\}$, $f^{-1}\{1,2,4\} = \{a,b,d\}$, $f^{-1}(\phi) = \phi$, $f^{-1}(Y) = X$.

Since the inverse image of each penta open set in Y under f is penta pre-open set in X . Hence f is penta pre-continuous function.

Theorem 4.3: Let $f:(X,T_1,T_2,T_3,T_4,T_5)\rightarrow\left(Y,W_1,W_2,W_3,W_4,W_5\right)$ be a penta pre-continuous open function .If A is an penta pre-open set of X , then $f(A)$ is penta pre-open in Y .

Proof: First, let A be penta pre-open set in X .There exist an penta open set U in X such that $A\subset U\subset p-cl(A)$ .since f is penta open function then $f(U)$ is penta open in Y . Since f is penta continuous function, we have $f(A)\subset f(U)\subset f(p-cl(A))\subset p-cl(f(A))$ .This show that $f(A)$ is penta pre-open in Y . Let A be penta pre-open in X .There exist an penta pre-open set U such that $U\subset A\subset (p-cl(U))$. Since f is penta-continuous function, we have by the proof of first part, $f(U)$ is penta pre-open in X .Therefore, $f(A)$ is penta pre-open in Y .

Theorem 4.4: Let $f:(X,T_1,T_2,T_3,T_4,T_5)\rightarrow\left(Y,W_1,W_2,W_3,W_4,W_5\right)$ be a penta pre-continuous open function .If V is an penta pre-open set of Y , then $f^{-1}(V)$ is penta pre-open in X .

Proof : First ,let V be penta pre-open set of Y .There exist an penta open set W in Y .such that $V\subset W\subset p-cl(V)$ .Since f is penta open set ,we have $f^{-1}(V)\subset f^{-1}(W)\subset f^{-1}(p-cl(V))\subset p-cl(f^{-1}(V))$ .since f is penta pre-continuous function , $f^{-1}(W)$ is penta pre-open set in X .By theorem 3.4 , $f^{-1}(V)$ is penta pre-open set in X .The proof of the second part is shown by using the fact of first part.

Theorem 4.5: The following are equivalent for a function $f:(X,T_1,T_2,T_3,T_4,T_5)\rightarrow\left(Y,W_1,W_2,W_3,W_4,W_5\right)$

a) f is penta pre-continuous function ;
b) the inverse image of each penta closed set of Y is penta pre-closed in X ;
c) For each $x\in X$ and each penta open set V in W containing $f(x)$,there exist an penta pre-open set U of X containing x such that $f(U)\subset V$;
d) $p-pcl(f^{-1}(B))\subset f^{-1}(p-cl(B))$ for every subset B of Y .
e) $f(p-pcl(A))\subset p-cl(f(A))$ for every subset A of X .

Theorem 4.6 : If $f:(X,T_1,T_2,T_3,T_4,T_5)\to(Y,W_1,W_2,W_3,W_4,W_5)$ and $g:(Y,W_1,W_2,W_3,W_4,W_5)\to(Z,\eta_1,\eta_2,\eta_3,\eta_4,\eta_5)$ be two penta pre-continuous function then $gof:(X,T_1,T_2,T_3,T_4,T_5)\to(Z,\eta_1,\eta_2,\eta_3,\eta_4,\eta_5)$ may not be penta pre-continuous function.

Theorem 4.7: Let $f^{-1}:(X,T_1,T_2,T_3,T_4,T_5)\to\left(Y,W_1,W_2,W_3,W_4,W_5\right)$ be bijective. Then the following conditions are equivalent:

i) f is a penta pre- open continuous function.

ii) f is penta pre-closed continuous function and

iii) f^{-1} is penta pre-continuous function.

Proof:(i)$\to$(ii) Suppose B is a penta closed set in X .Then $X-B$ is an penta open set in X .Now by (i) $f(X-B)$ is a penta pre-open set in Y .Now since f^{-1} is bijective so $f(X-B)=Y-f(B)$.Hence $f(B)$ is a penta pre-closed set in Y .Therefore f is a penta pre-closed continuous function.

(ii)$\to$(iii) Let f is an penta pre-closed map and B be penta closed set of X .Since f^{-1} is bijective so $(f^{-1})^{-1}(B)$ which is an penta pre-closed set in Y . Hence f^{-1} is penta pre-continuous function.

(iii)$\to$(i) Let A be a penta open set in X .Since f^{-1} is a penta pre-continuous function so $(f^{-1})^{-1}(A)=f(A)$ is a penta pre-open set in Y . Hence f is penta pre-open continuous function.

Theorem 4.8: Let X and Y are two penta topological spaces. Then $f:(X,T_1,T_2,T_3,T_4,T_5)\to\left(Y,W_1,W_2,W_3,W_4,W_5\right)$ is penta semi-continuous function if one of the followings holds:

i) $f^{-1}(p-\text{int}(B))\subseteq p-s\,\text{int}(f^{-1}(B))$, for every penta open set B in Y .

ii) $p-scl(f^{-1}(B))\subseteq f^{-1}(p-scl(B))$, for every penta open set B in Y .

Proof: Let B be any T open set in Y and if condition (i) is satisfied then $f^{-1}(p-s\,\text{int}(B))\subseteq p-s\,\text{int}(f^{-1}(B))$.

We get $f^{-1}(B) \subseteq p-s\,\text{int}(f^{-1}(B))$.Therefore $f^{-1}(B)$ is a penta semi-open set in X.Hence f is penta semi-continuous function. Similarly we can prove (ii).

Theorem 4.9: A function $f:(X,T_1,T_2,T_3,T_4,T_5) \to (Y,W_1,W_2,W_3,W_4,W_5)$ is called penta semi-open continuous function if and only if $f(p-s\,\text{int}(A)) \subseteq p-s\,\text{int}(f(A))$, for every penta-open set A in X.

Proof: Suppose that f is a penta semi-open continuous function.

since $p-s\,\text{int}(A) \subseteq A$ so $f(p-s\,\text{int}(A)) \subseteq f(A)$.

By hypothesis $f(p-s\,\text{int}(A))$ is an penta semi-open set and $p-s\,\text{int}(f(A))$ is largest penta semi-open set contained in $f(A)$ so $f(p-s\,\text{int}(A)) \subseteq p-s\,\text{int}(f(A))$.

Conversely, suppose A is an penta open set in X.So $f(p-s\,\text{int}(A)) \subseteq p-s\,\text{int}(f(A))$.

Now since $A = p-s\,\text{int}(A)$ so $f(A) \subseteq p-s\,\text{int}(f(A))$.Therefore $f(A)$ is a penta semi-open set in Y and f is penta semi-open continuous function.

Theorem 4.10: A function $f:(X,T_1,T_2,T_3,T_4,T_5) \to (Y,W_1,W_2,W_3,W_4,W_5)$ is called penta semi - closed continuous function if and only if $p-scl(f(A)) \subseteq f(p-s\,cl(A))$, for every penta closed set A in X.

Proof: Suppose that f is a penta semi-closed continuous function. since $A \subseteq p-scl(A)$ so $f(A) \subseteq f(p-scl(A))$.By hypothesis, $f(p-scl(A))$ is a penta semi-closed set and $p-scl(f(A))$ is smallest penta semi-closed set containing $f(A)$ so $p-scl(f(A)) \subseteq f(p-scl(A))$.

Conversely, suppose A is an penta closed set in X.So $p-scl(f(A)) \subseteq f(p-scl(A))$.

Since $A = p-scl(A)$ so $p-scl(f(A)) \subseteq f(A)$.Therefore $f(A)$ is a penta semi-closed set in Y and f is penta semi-closed continuous function.

CONCLUSION:

In this paper the idea of penta semi-open sets, penta semi-continuous function, penta pre-open sets and penta pre-continuous function in penta topological spaces were studied .

REFERENCES

[1] Jelic M. (1990), A decomposition of pairwise continuity. J. Inst. Math. Comput. Sci Math. Ser. 3, 25-29.

[2] Kelly J.C (1963.), Bitopological spaces, Proc. LondonMath.Soc.,3 PP. 17-89 .

[3] Khan M. S. and Khan G. A.(2018), p-Continuity and p-Homeomorphism in Penta Topological Spaces, European International Journal of Science and Technology, Vol. 7 ,No. 5, 1-8.

[4] Kovar M.,(2000).On 3-Topological version of Thet- Reularity, Internat. J. Matj, Sci., 23(6), 393-398.

[5] Levine N.(1963). Semi-open sets and semi-continuity in topological spaces, Amer. Math., 70, 36-41.

[6] Maheshwari S.N. & Prasad R. (1977). Semi-open sets and semi-continuous functions in bitopological spaces. Math. Notae. 26, 29-37.

[7] Mashhour A.S.,Abd El-Monsef, M.E. and El. Deep. S. N.(1981), On precontinuous and weak Precontinous mappings, Proc. Math. Phys. Soc. Egyp , vol 53,47-53.

[8] Mukundan D.V.(2013), Introduction to penta topological spaces, Int. Journal of Scientific and Engg. Research, 4(7) 2483-2485.

[9] Pacifica G. P & Fathima S.S.(2019), Some Topological Concepts in Penta Topological space, International Journal of Mathematics Trends and Technology, Volume 65 Issue 2- February, 109-116.

[10] Palaniammal S.(2011),Study of Tri topological spaces, Ph. D Thesis.