ON ALPHA ^ GENERALIZED CLOSED SETS IN TOPOLOGICAL SPACES

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ABSTRACT

The aim of this paper is to introduce a new class of sets called α^g - closed sets in topological spaces and to study their properties. Further, we define and study α^g - open sets α^g - continuity.

Key Words: α^g - closed sets, α^g - open sets, α^g continuous.

1. INTRODUCTION


In this paper, we introduce a new class of sets called alpha ^ generalized - closed sets (briefly α^g - closed sets) and we study their basic properties. We recall the following definitions, which will be used often throughout this paper.

2. PRELIMINARIES

Throughout this paper, X, Y, Z denote the topological spaces (X, τ),(Y, σ) and (Z, η) respectively, on which no separation axioms are assumed.

Definition 2.1: A subset A of a space X is called
(1) a pre-open set if A ⊆ int(cl(A)) and a pre-closed set if cl(int(A)) ⊆ A.
(2) a semi-open set if A ⊆ cl(int(A)) and a semi-closed set if int(cl(A)) ⊆ A.
(3) an α-open set if A⊆int(cl(int(A))) and a α-closed set if cl(int(cl(A)))⊆ A.
(4) a semi-preopen set (=β-open) if A ⊆ cl(int(cl(A))) and a semi-pre closed set (β-closed ) if int(cl(int(A))) ⊆ A.

The semi-closure (resp. α-closure) of a subset A of (X,τ) is denoted by scl(A) (resp. αcl(A) and spcl(A))and is the intersection of all semi-closed (resp. α-closed and semi-pre closed) sets containing A.
Definition 2.2: A subset $A$ of $X$ is called
1. a generalized closed (briefly $g$-closed) [9] set iff $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.
2. Strongly generalized closed (briefly $g^*$-closed) [20] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is g-open in $X$.
3. a regular open [18] set if $A = \text{int}(\text{cl}(A))$ and regular closed [18] set if $A = \text{cl}(\text{int}(A))$.
4. a semi generalized closed (briefly $sg$-closed) [4] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is Semiopen in $X$.
5. a generalized semi closed (briefly $gs$-closed) [2] if $\text{scl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.
6. a generalized semi-pre closed (briefly $gsp$-closed) [5] if $\text{spcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.
7. a regular generalized closed (briefly $rg$-closed) [15] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $X$.
8. a generalized preclosed (briefly $gp$-closed) [10] if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.
9. a generalized pre regular closed (briefly $gpr$-closed) [7] if $\text{pcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $X$.
10. a weakly closed (briefly $w$-closed) [16] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semiopen in $X$.
11. a regular weakly closed (briefly $rw$-closed) [3] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular semiopen in $X$.
12. a weakly generalized semi closed (briefly $wg$-closed) [13] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $X$.
13. a regular weakly generalized semi closed (briefly $rwg$-closed) [13] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $X$.
14. a regular generalized weakly semi closed (briefly $rgw$-closed) [17] if $\text{cl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular semi-open in $X$.
15. a regular generalized closed (briefly $r^g$-closed) [22] if $\text{gcl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open.
16. $g^*$ - closed set [23] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is semi-open in $(X, \tau)$.
17. $ag^*$ - closed set [24] if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g^*$-open in $(X, \tau)$.
18. $ag^*$ - closed set [25] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open in $(X, \tau)$.
19. $sg^*$ - closed set [26] if $\text{acl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g^*$-open in $(X, \tau)$.
20. $wg^*$ - closed set [27] if $\text{acl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open in $(X, \tau)$.
21. $wgg^*$ - closed set [27] if $\text{acl}(\text{int}(A)) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.
22. $\psi^*$ - closed set [28] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\psi$-open in $(X, \tau)$.
23. $\psi^*$ - closed set [29] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.
24. $g^*\psi^*$ - closed set [30] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g^*$-open in $(X, \tau)$.
25. $\psi^*$ - closed set [31] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $g^*$-open in $(X, \tau)$.
26. $\omega\psi$ - closed set [32] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open in $(X, \tau)$.
27. $g\omega^*$ - closed set [32] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open in $(X, \tau)$.
28. $\omega^*$ - closed set [33] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is open in $(X, \tau)$.
29. $g\omega$ - closed set [33] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is $\alpha$-open in $(X, \tau)$.
30. $g^*\omega$ - closed set [35] if $\text{cl}(A) \subseteq U$ whenever $A \subseteq U$ and $U$ is regular open in $(X, \tau)$. 
The complements of the above mentioned closed sets are their respective open sets.

**Definition 2.3:** A map \( f : X \rightarrow Y \) is said to be
1. a continuous function\([1]\) if \( f^{-1}(V) \) is closed in \( X \) for every closed set \( V \) in \( Y \).
2. a pre continuous \([11]\) if \( f^{-1}(V) \) is pre closed in \( X \) for every closed set \( V \) in \( Y \).
3. a \( \alpha \)-continuous function\([34]\) if \( f^{-1}(V) \) is \( \alpha \)-closed in \( X \) for every closed set \( V \) in \( Y \).
4. a gs -continuous \([1]\) if \( f^{-1}(V) \) is gs closed in \( X \) for every closed set \( V \) in \( Y \).
5. a \( \alpha g \)-continuous \([7]\) if \( f^{-1}(V) \) is \( \alpha g \) - closed in \( X \) for every closed set \( V \) in \( Y \).
6. a rwg-continuous\([13]\) if \( f^{-1}(V) \) is rwg- closed in \( X \) for every closed set \( V \) in \( Y \).
7. a rgw-continuous\([13]\) if \( f^{-1}(V) \) is rgw- closed in \( X \) for every closed set \( V \) in \( Y \).
8. a swg –continuous\([13]\) if \( f^{-1}(V) \) is swg- closed in \( X \) for every closed set \( V \) in \( Y \).

3. **Alpha ^ Generalized Closed Sets** (\( \alpha^g \) - closed sets)

**Definition 3.1:** A subset \( A \) of \((X, \tau)\) is called a \( \alpha^g \)generalized closed (briefly \( \alpha^g \) closed)
if \( \text{gcl}(A) \subset U \), whenever \( A \subset U \) and \( U \) is \( \alpha \)-open in \( X \).
We denote the family of all \( \alpha^g \) closed sets in space \( X \) by \( \alpha^GC(X) \).

**Theorem 3.2:** Every closed set of a topological space \((X, \tau)\) is \( \alpha^g \) closed set.

**Proof:** Let \( A \subset X \) be a closed set and \( A \subset U \) where \( U \) be \( \alpha \)-open. Since \( A \) is closed and every closed set is g-closed, \( \text{gcl}(A) \subset \text{cl}(A) = A \subset U \). Hence \( A \) is an \( \alpha^g \) closed set.

**Remark 3.3:** The converse of the above theorem need not be true as seen in the following example.

**Example 3.4:** Let \( X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, b\}\} \). Let \( A = \{a, c\} \) then \( A \) is an \( \alpha^g \) closed set but it is not a closed set.

**Theorem 3.5:** Every g^\alpha -closed set is \( \alpha^g \) closed.

**Proof:** Let \( A \) be a g^\alpha -closed set. Let \( A \subset U \) where \( U \) is \( \alpha \)-open. Since every \( \alpha \)-open set is semi open and \( A \) is g^\alpha closed, \( \text{cl}(A) \subset U \). Every closed set is g -closed therefore \( \text{gcl}(A) \subset \text{cl}(A) \subset U \). Hence \( A \) is \( \alpha^g \) closed.

**Remark 3.6:** The converse of the above theorem need not be true as seen in the following example.

**Example 3.7:** Let \( X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, b\}\} \). Let \( A = \{a, c\} \) then \( A \) is \( \alpha^g \) closed set but it is not a g^\alpha -closed set.

**Theorem 3.8:** Every \( \alpha^g \) closed set \( \alpha g \) is closed.

**Proof:** Let \( A \) be \( \alpha^g \) closed. Let \( A \subset U \) and \( U \) be open. Since every open set is \( \alpha \)-open set and \( A \) is \( \alpha^g \) closed set, \( \text{acl}(A) \subset U \). Hence \( A \) is \( \alpha g \) closed.

**Remark 3.9:** The converse of the above theorem need not be true as seen in the following example.

**Example 3.10:** Let \( X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, b\}\} \). Let \( A = \{b\} \) then \( A \) is \( \alpha g \) -closed set but it is not a \( \alpha^g \) -closed set.

**Theorem 3.11:** Every g^*closed set is \( \alpha^g \) closed.
Proof: Let \( A \) be \( g^* \)-closed in \((X, \tau)\). Let \( A \subseteq U \) where \( U \) is \( \alpha \)-open. Since every \( \alpha \)-open set is \( g^* \)-open and \( A \) is \( g^* \)-closed, \( \text{cl}(A) \subseteq U \). Every closed set is \( g \)-closed, then \( g\text{cl}(A) \subseteq \text{cl}(A) \subseteq U \). Hence \( A \) is \( \alpha^g \)-closed.

Remark 3.12: The converse of the above theorem need not be true as seen in the following example.

Example 3.13: Let \( X = \{a, b, c\}, \tau = \{X, \phi, \{a\}\} \). Let \( A = \{a, c\} \) then \( A \) is \( \alpha^g \)-closed set but it is not a \( g^* \)-closed set.

Theorem 3.14: Every \( ag^* \)-closed set is \( \alpha^g \)-closed.

Let \( A \) be \( ag^* \)-closed in \((X, \tau)\). Let \( A \subseteq U \) where \( U \) is \( \alpha \)-open. Since \( A \) is \( ag^* \)-closed, \( \text{cl}(A) \subseteq U \). Every closed set is \( g \)-closed, then \( g\text{cl}(A) \subseteq \text{cl}(A) \subseteq U \). Hence \( A \) is \( \alpha^g \)-closed.

Remark 3.15: The converse of the above theorem need not be true as seen in the following example.

Example 3.16: Let \( X = \{a, b, c\}, \tau = \{X, \phi, \{a\}\} \). Let \( A = \{a, b\} \) then \( A \) is \( \alpha^g \)-closed set but it is not a \( ag^* \)-closed set.

Example 3.17: Let \( X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, b\}\} \). Then
1. \( r\text{wg} \)-closed \( = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\} \)
2. \( r\text{g} \)-closed \( = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\} \)
3. \( \alpha g^* \)-closed \( = \{X, \phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}\} \)
4. \( \psi g^* \)-closed \( = \{X, \phi, \{b\}, \{c\}, \{a, c\}, \{b, c\}\} \)
5. \( r\text{rg} \)-closed \( = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\} \)
6. \( r\text{rg} \)-closed \( = \{X, \phi, \{a\}, \{b\}, \{c\}, \{a, b\}, \{a, c\}, \{b, c\}\} \)
7. \( \alpha^g \)-closed \( = \{X, \phi, \{c\}, \{a, c\}, \{b, c\}\} \)

Theorem 3.18:
1. Every \( \alpha^g \)-closed set is \( r\text{wg} \)-closed.
2. Every \( \alpha^g \)-closed set is \( r\text{g} \)-closed.
3. Every \( \alpha^g \)-closed set is \( -\alpha g^* \)-closed.
4. Every \( \alpha^g \)-closed set is \( -\psi g^* \)-closed.
5. Every \( \alpha^g \)-closed set is \( -\psi g^* \)-closed.
6. Every \( \alpha^g \)-closed set is \( -\alpha^g \)-closed.

Proof: Straight forward.

Remark 3.19: The converse of the above theorem need not be true as seen in the following examples.

In Example 3.14, \( A = \{a, b\} \), then \( A \) is \( r\text{wg} \)-closed but not \( \alpha^g \)-closed.
In Example 3.14, \( B = \{b\} \), then \( B \) is \( r\text{g} \)-closed but not \( \alpha^g \)-closed.
In Example 3.14, \( B = \{b\} \), then \( B \) is \( \psi g^* \)-closed but not \( \alpha^g \)-closed.
In Example 3.14, \( B = \{b\} \), then \( B \) is \( \psi g^* \)-closed but not \( \alpha^g \)-closed.
In Example 3.14, \( B = \{a\} \), then \( B \) is \( \alpha^g \)-closed but not \( \alpha^g \)-closed.
Remark 3.20: $\alpha^g$ closed sets and semi closed sets are independent to each other as seen from the following examples.

Example 3.21:
* Let $X = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Let $A = \{b\}$, $A$ is semi closed but not $\alpha^g$ closed and the subset $\{a, b, d\}$ in $X$ is $\alpha^g$ closed but not semi closed.

Remark 3.22: $\alpha^g$ closed sets and pre closed sets are independent to each other as seen from the following examples.

Example 3.23:
* Let $X = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Let $A = \{a\}$, then $A$ is $\alpha^g$ closed but not pre closed.
* Let $X = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Let $A = \{c\}$, then $A$ is preclosed but not an $\alpha^g$ closed set.

Remark 3.24: $\alpha^g$ closed sets and semi-preclosed sets are independent to each other as seen from the following example.

Example 3.25:
Let $X = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. The subset $\{a\}$ is semi- preclosed but not $\alpha^g$ closed and the subset $\{a, b, d\}$ is $\alpha^g$ closed but not semi- preclosed.

Remark 3.26: $\alpha^g$ closed sets and $g\alpha^*$ closed sets are independent to each other as seen from the following examples.

Example 3.27:
Let $X = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Let $A = \{c\}$, then $A$ is $g\alpha^*$ closed but not an $\alpha^g$ closed set in $X$ and the subset $\{a, d\}$ is an $\alpha^g$ closed set but not a $g\alpha^*$ closed set in $X$.

Remark 3.28: The concepts of $\alpha^g$ closed sets and $\alpha\psi$ closed sets are independent of each other as seen from the following examples.

Example 3.29:
Let $X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, b\}\}$. Let $B = \{b\}$, then $B$ is $\alpha\psi$ closed but it is not an $\alpha^g$ closed set and the subset $\{a, c\}$ is an $\alpha^g$ closed set but not $\alpha\psi$ closed set.

Remark 3.30: The concepts of $\alpha^g$ closed sets and $sg$ closed sets are independent of each other as seen from the following examples.

Example 3.31:
Let $X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, b\}\}$. Let $B = \{b\}$, then $B$ is $sg$ closed but it is not an $\alpha^g$ closed set and the subset $\{a, c\}$ is an $\alpha^g$ closed set but not $sg$ closed set.

Remark 3.32: The concepts of $\alpha^g$ closed sets and $\alpha$ closed sets are independent of each other as seen from the following examples.

Example 3.33:
* Let $X = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\}$. Let $A = \{a, b, d\}$, then $A$ is $\alpha^g$ closed but not $\alpha$-closed.
* Let \( X = \{a, b, c, d\}, \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\} \). Let \( A = \{c\} \), then \( A \) is \( \alpha \)-closed but not an \( \alpha^g \)-closed set.

**Remark 3.34:** \( \alpha^g \)-closed sets and \( s\alpha^g \)-closed sets are independent to each other as seen from the following examples.

**Example 3.35:**
Let \( X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, b\}\} \). The subset \( \{a, b\} \) is \( s\alpha^g \)-closed set but not an \( \alpha^g \)-closed set and the subset \( \{b, c\} \) is \( \alpha^g \)-closed set but not \( s\alpha^g \)-closed.

**Remark 3.36:** \( \alpha^g \)-closed sets and \( w\alpha \)-closed sets are independent to each other as seen from the following examples.

**Example 3.37:**
Let \( X = \{a, b, c\}, \tau = \{X, \phi, \{a\}, \{a, b\}\} \). The subset \( \{b\} \) is \( w\alpha \)-closed set but not an \( \alpha^g \)-closed set and the subset \( \{a, c\} \) is \( \alpha^g \)-closed set but not \( w\alpha \)-closed.

**Remark 3.38:** The above discussions are shown in the following diagram.

**Remark 3.39:** The following is the diagrammatic representation of independent concepts of the sets with \( \alpha^g \)-closed sets.

**Theorem 3.40:** Let \( A \) be an \( \alpha^g \)-closed set in a topological space \( X \). Then \( gcl(A) \) -- \( A \) contains no non-empty \( \alpha \)-closed set in \( X \).

**Proof:** Let \( F \) be a \( \alpha \)-closed set such that \( F \subset gcl(A) \) -- \( A \). Then \( F \subset X-A \) implies \( A \subset X-F \). Since \( A \) is \( \alpha^g \)-closed and \( X-F \) is \( \alpha \)-open, then \( gcl(A) \subset X-F \). That is \( F \subset X-gcl(A) \). Hence \( F \subset \)
gcl(A) \cap (X \setminus \text{gcl}(A)) = \phi. \text{ Thus } F = \phi, \text{ whence gcl}(A) - A \text{ does not contain nonempty } \alpha\text{-closed set.}

**Remark 3.41:** The converse of the above theorem need not be true, that means if gcl(A) - A contains no nonempty \( \alpha \)-closed set, then A need not to be an \( \alpha^g \) closed as seen in the following example.

**Example 3.42:**

Let \( X = \{a, b, c, d\} \), \( \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}, \{a, b, c\}\} \). Let A = \{b\}. gcl(A) - A = \{d\}, it does not contain non-empty \( \alpha \)-closed set in X. But A = \{b\} is not an \( \alpha^g \) closed set.

**Theorem 3.43:** The finite union of two \( \alpha^g \) closed sets are \( \alpha^g \) closed.

**Proof:** Assume that A and B are \( \alpha^g \) closed sets in X. Let A \( \cup \) B \( \subset \) U where U is \( \alpha \)-open. Then A \( \subset \) U and B \( \subset \) U. Since A and B are \( \alpha^g \) closed, gcl(A) \( \subset \) U and gcl(B) \( \subset \) U. Then gcl(A \( \cup \) B) = gcl(A) \( \cup \) gcl(B) \( \subset \) U. Hence A \( \cup \) B is \( \alpha^g \) closed.

**Remark 3.44:** The intersection of two \( \alpha^g \) closed set in X need not be an \( \alpha^g \) closed set as seen in the following example.

**Example 3.45:**

Let \( X = \{a, b, c\} \), \( \tau = \{X, \phi, \{a\}, \{b\}\} \) then \( \alpha^g = \{X, \phi, \{b\}, \{a, b\}, \{a, c\}, \{b, c\}\} \).

If A = \{a, b\} and B = \{a, c\}. Then A and B are \( \alpha^g \) closed sets. But A \( \cap \) B = \{a\} is not an \( \alpha^g \) closed set.

**Theorem 3.46:** If A is an \( \alpha^g \) closed subset of X such that A \( \subset \) B \( \subset \) gcl(A), then B is an \( \alpha^g \) closed set.

**Proof:** Let B \( \subset \) U where U is \( \alpha \)-open. Then A \( \subset \) B implies A \( \subset \) U. Since A is \( \alpha^g \) closed, gcl(A) \( \subset \) U. By hypothesis gcl(B) \( \subset \) gcl(gcl(A)) = gcl(A) \( \subset \) U. Hence B is \( \alpha^g \) closed.

**Remark 3.47:** The converse of the above theorem need not be true as seen in the following example.

**Example 3.48:**

Let X = \{a, b, c, d\}, \( \tau = \{X, \phi, \{a\}, \{b\}, \{a, b\}\} \). Let A = \{d\} and B = \{a, d\}. Then A and B are \( \alpha^g \) closed sets. But A \( \cap \) B = \{a\} is not a subset of gcl(A).

4. Regular \( \wedge \) Generalized Open Set:

**Definition 4.1:** A set A \( \subset \) X is called alpha \( \wedge \) generalized open (\( \alpha^g \) open) set if and only if its compliment is alpha \( \wedge \) generalized closed. The collection of all \( \alpha^g \) open sets is denoted by \( \alpha^G(X) \).

5. \( \alpha^g \) Continuous and \( \alpha^g \) Irresolute Functions:

**Definition 5.1:** A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called \( \alpha^g \) continuous if every \( f^{-1}(V) \) is \( \alpha^g \) closed in X for every closed set V of Y.

**Definition 5.2:** A function \( f : (X, \tau) \rightarrow (Y, \sigma) \) is called \( \alpha^g \) irresolute if every \( f^{-1}(V) \) is \( \alpha^g \) closed in X for every \( \alpha^g \) closed set V of Y.

**Example 5.3:** Let X = \{a, b, c\}, \( \tau = \{X, \phi, \{a\}\} \) and Y = \{a, b\},
\[ \sigma = \{ Y, \phi, \{ a \} \} \]. Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = a, f(b) = b, f(c) = c \). Here the inverse image of the closed sets in \( Y \) are \( \alpha^g \) closed sets in \( X \). Hence \( f \) is \( \alpha^g \) continuous.

**Example 5.4:** Let \( X = \{ a, b, c, d \} \), \( \tau = \{ \phi, X, \{ a, b \}, \{ c \}, \{ a, b, c \} \} \) and \( Y = X \), \( \sigma = \{ Y, \phi, \{ a \}, \{ c \}, \{ a, c \}, \{ a, b \}, \{ a, b, c \} \} \). Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) by \( f(a) = a, f(b) = b, f(c) = c, f(d) = d \). The inverse image of every \( \alpha^g \) closed set in \( Y \) is \( \alpha^g \) closed set in \( X \). Hence \( f \) is \( \alpha^g \) continuous.

**Remark 5.5:** Every \( \alpha^g \) irresolute function is \( \alpha^g \) continuous but the converse is not true as seen in the following example.

**Example 5.6:** In example 5.3, \( f \) is \( \alpha^g \) continuous but not \( \alpha^g \) irresolute.

**Remark 5.7:** Every continuous function is \( \alpha^g \) continuous. But the converse is not true as seen in the following example.

**Example 5.8:** Let \( X = \{ a, b, c, d \} \), \( \tau = \{ X, \phi, \{ a \}, \{ b \}, \{ a, b \}, \{ a, b, c \} \} \) and \( Y = X \), \( \sigma = \{ Y, \phi, \{ a \}, \{ c \}, \{ a, c \}, \{ a, b \}, \{ a, c, d \} \} \). Define \( f : (X, \tau) \rightarrow (Y, \sigma) \) the identity mapping. Then \( f \) is \( \alpha^g \) continuous but not \( \alpha^g \) irresolute.

**Theorem 5.9:** Let \( f : (X, \tau) \rightarrow (Y, \sigma) \) and \( g : (Y, \sigma) \rightarrow (Z, \eta) \) be any two functions. Then

(i) \( \alpha^g \) continuous if \( g \) is continuous and \( f \) is \( \alpha^g \) -continuous

(ii) \( \alpha^g \) irresolute if \( g \) is \( \alpha^g \) irresolute and \( f \) is \( \alpha^g \) irresolute.

(iii) \( \alpha^g \) continuous if \( g \) is \( \alpha^g \) continuous and \( f \) is \( \alpha^g \) irresolute.

**Proof:**

(i) Let \( V \) be any closed set in \( (Z, \eta) \). Then \( f^{-1}(V) \) is closed in \( (Y, \sigma) \), since \( g \) is continuous. By hypothesis \( f^{-1}(g^{-1}(V)) \) is \( \alpha^g \) closed in \( (X, \tau) \). Hence \( g \) is \( \alpha^g \) continuous.

(ii) Let \( V \) be \( \alpha^g \) closed set in \( (Z, \eta) \). Since \( g \) is \( \alpha^g \) irresolute, \( g^{-1}(V) \) is \( \alpha^g \) closed in \( (Y, \sigma) \). As \( f \) is \( \alpha^g \) irresolute, \( f^{-1}(g^{-1}(V)) = (gof)^{-1}(V) \) is \( \alpha^g \) closed in \( (X, \tau) \). Hence \( g \) is \( \alpha^g \) irresolute.

(iii) Let \( V \) be closed in \( (Z, \eta) \). Since \( g \) is \( \alpha^g \) continuous, \( g^{-1}(V) \) is \( \alpha^g \) closed in \( (Y, \sigma) \). As \( f \) is \( \alpha^g \) irresolute, \( f^{-1}(g^{-1}(V)) = (gof)^{-1}(V) \) is \( \alpha^g \) closed in \( (X, \tau) \). Hence \( g \) is \( \alpha^g \) continuous.

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