Modeling and Stability Analysis Of SIQS Cholera Transmission Dynamics

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Abstract

The present paper deals with a Susceptible-Infected-Quarantined-Susceptible mathematical model for cholera transmission dynamics. Find out disease-free and endemic equilibria and basic reproduction number $R_q$. Routh-Hurwitz criteria, Castillo-Chavez criteria and Dulac’s criterion plus Poincare-Bendixson theorem have been applied for analyzing stability for the considered model. Numerical simulations are also conceded.

Keywords: Quarantine, Reproduction number, Routh–Hurwitz criteria, Castillo-Chavez criteria.

MSC: 34D20, 49J15, 92D25, 92D30, 93D20.

1. Introduction

Adebimpe O. et.al. [1] studied an SIQS epidemic model with saturated incidence rate and discussed the stability of disease-free and endemic equilibrium using different criterias. Castillo-Chavez et. al. discussed reproductive number and stability for some epidemic models. Specially, they gave a theorem on globally analysis of equilibrium points known as Castillo-Chavez criterion[2]. An SIS cholera epidemic model with quarantine effect have been developed by Mokati D. et al.[3]. Many researchers investigated the composite behavior of cholera disease. Nirwani N. et al. proposed a SIQR-B cholera epidemic model and gave the results about the effects of quarantine for the cholera model[4]. Pang Y. et al. discussed the dynamics of a stochastic SIQS outbreak model [5].

2. The Mathematical model

We have considered an SIQS cholera dynamical model in which human population is divided into susceptible ($S$), infectious ($I$), quarantine ($Q$) and recovered ($R$) individuals at time $t$ and the pathogen population is assumed as $B(t)$ at time $t$. Now consider the total number of population at time $t$ is $S+I+Q+R=1$. 
The following figure 1 represents the flow of individuals for the considered model:

![Figure 1: Transfer diagram for SIQS cholera model]

The symbols are used here stands for:

- $\mu_1 =$ Natural human birth and death rate,
- $\beta_1, \beta_2 =$ Contact rates for the human-environment & human-human interactions respectively,
- $\alpha_1, \alpha_2 =$ Constant rates,
- $d_1, d_2 =$ Disease related death rate constant in $I$ & $Q$ respectively,
- $\alpha =$ Recovery rate from the disease,
- $\delta =$ Transmission rate between compartments $I$ to $Q$,
- $\varepsilon =$ Transmission rate between compartments $Q$ to $R$,
- $\eta =$ Rate of human contribution to the growth of the pathogen,
- $\mu_2 =$ Death rate of the pathogen in the environment,
- $\omega =$ Disease transmission rate from compartment $I$ to $S$.
- $\theta =$ Disease transmission rate between compartments $Q$ to $S$.

All parameters are assumed nonnegative.

3. Mathematical Analysis of the model

The differential equations corresponding to the transfer diagram are:

$$
\begin{align*}
\frac{dS}{dt} &= \mu_1 S - \mu_1 S - \beta_1 SB - \beta_2 SI / (1 + \alpha_1 B + \alpha_2 B) \\
\frac{dI}{dt} &= \beta_1 SB / (1 + \alpha_1 B + \alpha_2 B) - (\mu_1 + d_1) I - \delta I \\
\frac{dQ}{dt} &= \beta_2 SI / (1 + \alpha_1 B + \alpha_2 B) - (\mu_1 + d_2) Q - \varepsilon Q \\
\frac{dB}{dt} &= \eta I - \mu_2 B \\
\frac{dR}{dt} &= \delta I - \mu_1 R
\end{align*}
$$
The system (1) can also be considered in the form of system (2)

\[
\frac{dS}{dt} = \mu_1 + \omega I + \theta Q - \frac{\beta_1 S B}{1 + \alpha_1 B} - \frac{\beta_2 S I}{1 + \alpha_2 I} - \mu_i S
\]

\[
\frac{dI}{dt} = \frac{\beta_1 S B}{1 + \alpha_1 B} + \frac{\beta_2 S I}{1 + \alpha_2 I} - (d_i + \mu_i + \delta + \alpha + \omega) I
\]

\[
\frac{dQ}{dt} = \delta I - (\varepsilon + d_z + \mu_i + \theta) Q
\]

\[
\frac{dB}{dt} = \alpha I + \varepsilon Q - \mu_i R
\]

\[
= \eta I - \mu_a B
\]

The feasible region of human population \( D \) and pathogen \( \Omega \) corresponding to the system (2) will be

\[
D = \{(S, I, Q): S \geq 0, I \geq 0, Q \geq 0, S + I + Q \leq 1\}
\]

\[
\Omega = \{B: B \geq 0\}
\]

4. Equilibrium points

4.1 Disease-free equilibrium \( (E^0) \)

The system (2) can be written as

\[
\mu_1 + \omega I + \theta Q - \frac{\beta_1 S B}{1 + \alpha_1 B} - \frac{\beta_2 S I}{1 + \alpha_2 I} - \mu_i S = 0
\]

\[
\frac{\beta_1 S B}{1 + \alpha_1 B} + \frac{\beta_2 S I}{1 + \alpha_2 I} - (d_i + \mu_i + \delta + \alpha + \omega) I = 0
\]

\[
\delta I - (\varepsilon + d_z + \mu_i + \theta) Q = 0
\]

\[
\eta I - \mu_a B = 0
\]

Assume that \( I = 0 \) (no disease), then \( Q = 0 \) and \( B = 0 \) and \( S = 1 \). Thus, \( E^0 = (1, 0, 0, 0) \).
4.2 Endemic equilibrium ($E^*$)

Assume that there is disease occurs, then $I \neq 0$.

System (2) can be written as

$$\mu_i + \sigma I^* + \theta Q^* - \frac{\beta_i S^* B^*}{1 + \alpha_i B^*} - \frac{\beta_2 S^* I^*}{1 + \alpha_2 I^*} - \mu_i S^* = 0$$

$$\frac{\beta_i S^* B^*}{1 + \alpha_i B^*} + \frac{\beta_2 S^* I^*}{1 + \alpha_2 I^*} - (d_1 + \mu_i + \alpha + \delta + \omega) I^* = 0$$

$$\delta I^* - (d_2 + \mu_i + \epsilon + \theta) Q^* = 0$$

$$\eta I^* - \mu_2 B^* = 0$$

From equations third and fourth, we obtain

$$Q^* = \frac{\delta I^*}{(\epsilon + \mu_i + d_2 + \theta)}$$

respectively. On solving first and second equations, we get

$$S^* = \frac{1}{\mu_i} \left[ \mu_i - (d_1 + \mu_i + \alpha + \delta) I^* + \frac{\theta \delta I^*}{(\epsilon + \mu_i + d_2 + \theta)} \right]$$

Again, from first equation,

$$\left\{ \left[ \frac{\beta_i \eta}{\mu_2 + \alpha_i \eta I^*} + \frac{\beta_2}{1 + \alpha_2 I^*} \right] \frac{1}{\mu_i} \left[ \mu_i - (d_1 + \mu_i + \alpha + \delta) I^* + \frac{\theta \delta I^*}{(\epsilon + \mu_i + d_2 + \theta)} \right] - (d_1 + \mu_i + \alpha + \delta + \omega) \right\} I^* = 0.$$ 

But $I^* \neq 0$, so

$$\left\{ \left[ \frac{\beta_i \eta}{\mu_2 + \alpha_i \eta I^*} + \frac{\beta_2}{1 + \alpha_2 I^*} \right] \frac{1}{\mu_i} \left[ \mu_i - (d_1 + \mu_i + \alpha + \delta) I^* + \frac{\theta \delta I^*}{(\epsilon + \mu_i + d_2 + \theta)} \right] - (d_1 + \mu_i + \alpha + \delta + \omega) \right\} = 0.$$ 

Or

$$\frac{1}{\mu_i} \left[ \mu_i - (d_1 + \mu_i + \alpha + \delta) I^* + \frac{\theta \delta I^*}{(\epsilon + \mu_i + d_2 + \theta)} \right] = \frac{(d_1 + \mu_i + \alpha + \delta + \omega)}{\left[ \frac{\beta_i \eta}{\mu_2 + \alpha_i \eta I^*} + \frac{\beta_2}{1 + \alpha_2 I^*} \right].}$$

Take $g_1(I^*) = g_2(I^*)$, where

$$g_1(I^*) = \frac{1}{\mu_i} \left[ \mu_i - (d_1 + \mu_i + \alpha + \delta) I^* + \frac{\theta \delta I^*}{(\epsilon + \mu_i + d_2 + \theta)} \right]$$

and

$$g_2(I^*) = \frac{(d_1 + \mu_i + \alpha + \delta + \omega)}{\left[ \frac{\beta_i \eta}{\mu_2 + \alpha_i \eta I^*} + \frac{\beta_2}{1 + \alpha_2 I^*} \right].}$$

Now, presume that $I = I^*$, then
\[ g_1(I) = \frac{1}{\mu_1} \left[ \mu_i - (d_1 + \mu_i + \alpha + \delta) I + \frac{\theta \delta I}{(\varepsilon + \mu_i + d_2 + \theta)} \right] \quad \text{and} \quad g_2(I) = \frac{\left( d_1 + \mu_i + \alpha + \delta + \omega \right)}{\mu_2 + \alpha_1 \eta I + \frac{\beta_2}{1+\alpha_2 I}} \]

If \( I = 0 \), then \( g_1(0) = 1 \) and \( g_2(0) = \frac{\mu_i (d_1 + \mu_i + \alpha + \delta + \omega)}{(\beta_1 \eta + \beta_2 \mu_2)} \).

If \( I > 0 \) then \( g_1(I) < 0 \) and \( g_2(I) > 0 \).

Then, we observe that \( g_2(I) \) is a rising function for \( I \geq 0 \).

Hence, basic reproduction number \( R_q \) is given by \( R_q = \frac{(\beta_1 \eta + \beta_2 \mu_2)}{\mu_2 (d_1 + \mu_i + \alpha + \delta + \omega)} \).

If \( R_q > 1 \), then \( g_2(0) < 1 \).

Thus, \( E^* = \left( \frac{1}{\mu_1} \left[ \mu_i - (d_1 + \mu_i + \alpha + \delta) I^* + \frac{\theta \delta I^*}{(\varepsilon + \mu_i + d_2 + \theta)} \right], I^*, \frac{\delta I^*}{(\varepsilon + \mu_i + d_2 + \theta)}, \frac{\eta I^*}{\mu_2} \) \),

where \( I^* \) can be found out on solving \( g_1(I^*) = g_2(I^*) \).

5. Stability Analysis

**Theorem 1.** If \( R_q < 1 \), then the disease-free equilibrium is locally asymptotically stable.

**Proof.** Consider the matrix at disease-free equilibrium as

\[
J(E^0) = \begin{bmatrix}
-\mu_i & \omega - \beta_2 & \theta & -\beta_1 \\
0 & \beta_2 - (d_1 + \mu_i + \delta + \alpha + \omega) & 0 & \beta_1 \\
0 & \delta & -(\varepsilon + d_2 + \mu_i + \theta) & 0 \\
0 & \eta & 0 & -\mu_2 \end{bmatrix}
\]

Then,

\[ |J(E^0) - z I| = 0. \]

\[ \Rightarrow \begin{vmatrix}
-\mu_i - z & \omega - \beta_2 & \theta & -\beta_1 \\
0 & \beta_2 - (d_1 + \mu_i + \delta + \alpha + \omega + z) & 0 & \beta_1 \\
0 & \delta & -(\varepsilon + d_2 + \mu_i + \theta + z) & 0 \\
0 & \eta & 0 & -\mu_2 - z
\end{vmatrix} = 0. \]

On simplification, we have

\((\varepsilon + d_2 + \mu_i + \theta + z)(\mu_i + z)\{z^2 + (\mu_2 + d_1 + \mu_i + \delta + \alpha + \omega - \beta_2) z + \mu_2 (d_1 + \mu_i + \delta + \alpha + \omega) - (\mu_2 \beta_2 + \beta_1 \eta)\} = 0.\)

First two eigen values are \( z = -\mu_i < 0 \) and \( z = -(\varepsilon + \mu_i + d_2 + \theta) < 0. \)
Remaining two eigen values are obtaining by the following equation
\[ z^2 + (\mu_2 + d_1 + \mu_1 + \delta + \alpha + \omega - \beta_2)z + \mu_2 (d_1 + \mu_1 + \delta + \alpha + \omega) - (\mu_2 \beta_2 + \beta_1 \eta) = 0. \]
which can be written as
\[ z^2 + a_1 z + a_2 = 0 \]
where,
\[ a_1 = (\mu_2 + d_1 + \mu_1 + \delta + \alpha + \omega - \beta_2), \]
\[ a_2 = \mu_2 (d_1 + \mu_1 + \delta + \alpha + \omega) - (\mu_2 \beta_2 + \beta_1 \eta) = \mu_2 (d_1 + \mu_1 + \delta + \alpha + \omega) \left[ 1 - \frac{\mu_2 \beta_2 + \beta_1 \eta}{\mu_2 (d_1 + \mu_1 + \delta + \alpha + \omega)} \right] \]
= 1 - R_q.
where, \( R_q = \frac{(\beta_1 \eta + \beta_2 \mu_2)}{\mu_2 (d_1 + \mu_1 + \alpha + \delta + \omega)} < 1. \)

Hence, \( a_1 > 0, a_2 > 0 \) and \( a_1 a_2 > 0 \)
Thus , by the Routh-Hurwitz criteria ,the theorem is proved.

**Theorem 2.** The disease-free equilibrium is globally asymptotically stable if \( R_q < 1. \)

**Proof.** We employ method of Castillo-Chavez. System (1) can be consider into two compartments , that is , uninfected and infected individuals, given by \( a_1 : \frac{dU}{dt} = M(U,V) \)
and \( a_2 : \frac{dV}{dt} = P(U,V), P(U,0) = 0, \)
Where , \( U = (S,R) \in R^2, V = (I,Q,B) \in R^3. \)

Let \( E^0 = (N_0,0), N_0 = \left( \frac{\pi}{\mu + \nu} \right). \)

Then \( E^0 = (N_0,0) \) is globally asymptotically stable equilibrium of (5) if the following conditions are satisfied :
\( b_1 : E^0 \) is globally asymptotically stable for \( \frac{dU}{dt} = M(U,0), b_2 : P(U,V) \succeq 0, (U,V) \in D, \)
where \( P(U,V) = AV - \hat{P}(U,V) , A \) is a Metzier matrix.

Then we can write \( A = F - V. \) Now,
\[ \frac{dU}{dt} = M(U,V) = \begin{bmatrix} \pi - \frac{B \beta_1 S}{k + B} - \beta_1 SI - (\mu + \nu)S + \alpha Q \vspace{1mm} \\ \nu S + \eta Q - \mu R \end{bmatrix} \]
and \( b_1 : \frac{dU}{dt} = M(U,0) = \begin{bmatrix} \pi - (\mu + \nu)S \\
\nu S \end{bmatrix} \)
and \[\frac{dV}{dt} = P(U,V) = \begin{bmatrix} \frac{B\beta IS}{k+B} + \beta SI - (\mu + \delta + \gamma)I \\ \gamma I - (\mu + \delta + \eta + \alpha)Q \\ \varepsilon I - cB \end{bmatrix}, \quad P(U,0) = 0.\]

Hence, \(b_1\) is satisfied.

Now for \(b_2\), \(A = F - V = \begin{bmatrix} \frac{\beta \pi}{\mu + v} - (\mu + \delta + \gamma) & 0 & \frac{\beta \pi}{\mu + v} \\ \gamma & - (\mu + \delta + \eta + \alpha) & 0 \\ \varepsilon & 0 & -c \end{bmatrix}\) and

\[AV = \begin{bmatrix} \frac{\beta \pi}{\mu + v} - (\mu + \delta + \gamma) & 0 & \frac{\beta \pi}{\mu + v} \\ \gamma & - (\mu + \delta + \eta + \alpha) & 0 \\ \varepsilon & 0 & -c \end{bmatrix} \begin{bmatrix} I \\ Q \\ B \end{bmatrix} = \begin{bmatrix} \frac{\beta \pi I}{\mu + v} - (\mu + \delta + \gamma)I + \frac{\beta \pi B}{\mu + v} \\ \gamma I - (\mu + \delta + \eta + \alpha)Q \\ \varepsilon I - cB \end{bmatrix} \]

Thus, \(P(U,V) = AV - P(U,V) = \begin{bmatrix} 0 \\ 0 \\ 0 \end{bmatrix} = \begin{bmatrix} \hat{P}(U,V)_1 \\ \hat{P}(U,V)_2 \\ \hat{P}(U,V)_3 \end{bmatrix}\) Hence, \(b_2\) is satisfied.

This completes the proof.

**Theorem 3.** If \(R_q > 1\), then the endemic equilibrium is locally asymptotically stable.

**Proof.** The variational matrix will be

\[J(E) = \begin{bmatrix} \frac{-\beta \beta B^*}{1+\alpha B^*} - \frac{\beta \beta I^*}{1+\alpha I^*} - \mu_i & \omega - \frac{\beta \beta S^*}{\alpha ^2 I^2} & \theta & \frac{\beta \beta S^*}{\alpha ^2 I^2} \\ \omega - \frac{\beta \beta S^*}{\alpha ^2 I^2} & 0 & \frac{\beta \beta S^*}{\alpha ^2 I^2} \\ \frac{\beta \beta B^*}{1+\alpha B^*} + \frac{\beta \beta I^*}{1+\alpha I^*} & \frac{\beta \beta S^*}{\alpha ^2 I^2} & \frac{\beta \beta S^*}{\alpha ^2 I^2} \end{bmatrix} \]

where,

\[J(E^*) = \begin{bmatrix} -J_1 - \mu_i & \omega - J_2 & \theta & -J_3 \\ J_1 & K & 0 & J_3 \\ O & \delta & -L & O \\ O & \eta & 0 & -\mu_2 \end{bmatrix} \]
\[ J_1 = \frac{\beta_1 B^*}{1 + \alpha_1 B^*} + \frac{\beta_2 I^*}{1 + \alpha_2 I^*}, \quad J_2 = \frac{\beta_2 S^*}{(1 + \alpha_1 I^*)^2}, \quad J_3 = \frac{\beta_1 S^*}{(1 + \alpha_2 B^*)^2}, \]

\[ K = J_2 - (d_1 + \mu_1 + \alpha + \delta + \omega), \quad \text{and} \quad L = (\varepsilon + \mu_3 + d_2 + \theta). \]

Then,

\[ \left| J(E^*) - \lambda I \right| = 0. \]

\[ \Rightarrow \quad J(E^*) = \begin{vmatrix} -J_1 - \mu_4 - \lambda & \omega - J_2 & \theta & -J_3 \\ J_1 & K - \lambda & 0 & J_3 \\ 0 & \delta & -(L + \lambda) & 0 \\ 0 & \eta & 0 & -\mu_2 - \lambda \end{vmatrix} = 0. \]

On simplification,

\[ \lambda^4 + \lambda^3 (-K + L + \mu_2 + J_1 + \mu_4) + \lambda^2 (-K \mu_2 - KL - (J_1 + \mu_4) + (J_1 + \mu_4) \mu_3 + \mu_2 L - (J_1 + \mu_4) L - J_3 \eta - J_2 \omega - J_2 J_2) \]

\[ + \lambda ((J_1 + \mu_4) - (K(\mu_2 + L) + \mu_2 L - J_2 \eta) - K \mu_2 L - J_2 \eta L - J_1 \omega(\mu_2 + L) - J_2 (\mu_2 + L) - J_2 \eta + \theta \delta)) \]

\[ + (J_1 \omega(\mu_2 + L) + J_1 \delta \mu_2 - J_1 J_2 \mu_2 L - J_2 \eta L - (J_1 + \mu_4) \lambda K \mu_2 - (J_1 + \mu_4) J_3 L J_3 \eta L) = 0. \]

or

\[ \lambda^4 + a_1 \lambda^3 + a_2 \lambda^2 + a_3 \lambda + a_4 = 0 \]

where

\[ a_1 = (-K + L + \mu_2 + J_1 + \mu_4) = 3 \mu_4 + (d_1 + \delta + \alpha + \omega + \varepsilon + d_2 + \theta) + \mu_2 + J_1 - J_2 > 0 \]

\[ a_2 = (-K \mu_2 - KL - (J_1 + \mu_4) + (J_1 + \mu_4) \mu_2 L - (J_1 + \mu_4) L - J_3 \eta - J_1 \omega - J_2 J_2) \]

\[ = (J_1 + \mu_4) (2 \mu_4 + d_1 + \delta + \alpha + \omega + \varepsilon + d_2 + \theta + \mu_2 - J_2) - (J_1 + \mu_4 + d_1 + \delta + \alpha + \omega)) (-\varepsilon + d_2 + \theta + \mu_2 + \mu_4) + \mu_4 (\varepsilon + d_2 + \theta + \mu_2 + \mu_4) \]

\[ + \mu_2 (\varepsilon + d_2 + \theta + \mu_2) - J_3 \eta - J_1 \omega - J_2 J_2 > 0 \]

\[ a_3 = ((J_1 + \mu_4) - (K(\mu_2 + L) + \mu_2 L - J_2 \eta) - K \mu_2 L - J_3 \eta L + J_1 \omega(\mu_2 + L) - J_2 (\mu_2 + L) - J_3 \eta + \theta \delta)) \]

\[ = (J_1 + \mu_4) ((J_2 - (\mu_4 + d_1 + \delta + \alpha + \omega) (\varepsilon + d_2 + \theta + \mu_2 + \mu_4) + \mu_2 (\varepsilon + d_2 + \theta + \mu_2) - J_3 \eta)) \]

\[ - \mu_2 (J_2 - (\mu_4 + d_1 + \delta + \alpha + \omega) (\varepsilon + d_2 + \theta + \mu_2) - J_3 \eta (\varepsilon + d_2 + \theta + \mu_2) + J_1 \omega (\mu_2 + \varepsilon + d_2 + \theta + \mu_4) \]

\[ - J_2 (\mu_2 + \varepsilon + d_2 + \theta + \mu_2) J_3 \eta + \theta \delta) > 0 \]

\[ a_4 = (J_1 \omega(\mu_2 + L) + J_1 \delta \mu_2 - J_1 J_2 \mu_2 L - J_1 \eta L - (J_1 + \mu_4) \lambda K \mu_2 - (J_1 + \mu_4) J_3 L J_3 \eta L) \]

\[ = J_1 \omega(\mu_2 + L) + J_1 \delta \mu_2 - J_1 J_2 \mu_2 L - J_1 \eta L - (J_1 + \mu_4) \lambda K \mu_2 - (J_1 + \mu_4) J_3 L J_3 \eta L \]

\[ = (J_1 + \mu_4) (\varepsilon + d_2 + \theta + \mu_2) (J_2 - (\mu_4 + d_1 + \delta + \alpha + \omega)) - (J_1 + \mu_4) J_3 \eta (\varepsilon + d_2 + \theta + \mu_2) > 0 \]

It is clearly seen that \( a_1 > 0, a_2 > 0, a_3 > 0, a_4 > 0 \) and \( a_1 a_2 a_3 > a_2^2 + a_1^2 a_4 \) Hence, by

Routh-Hurwitz criteria, the theorem is proved.

**Theorem 4.** The endemic equilibrium is globally asymptotically stable if \( R_q > 1 \).

**Proof.** Assume
We use Dulac plus Poincare Bendixson theorem as follows

Consider , \( H(S, I, Q, R, B) = \frac{1}{SIQRB} \) where \( S > 0, I > 0, Q > 0, R > 0, B > 0 \).

Then ,

\[
\nabla(HF) = \frac{\partial}{\partial S} (H.F_1) + \frac{\partial}{\partial I} (H.F_2) + \frac{\partial}{\partial Q} (H.F_3) + \frac{\partial}{\partial R} (H.F_4) + \frac{\partial}{\partial B} (H.F_5)
\]

\[
\Rightarrow \nabla(HF) = -\frac{\pi}{S^2IQR} - \frac{\alpha}{S^2IR} - \frac{\beta_e}{I^2QR(k+B)} - \frac{\gamma}{SQ^2R} - \frac{\nu}{IQR^2B} - \frac{\eta}{SIR^2B} - \frac{\varepsilon}{SQR^2} < 0.
\]

Hence, by Dulac’s criterion, The proof is completed.

6. Numerical Simulation and graphical representation

Case I: Disease dies out at \( R_q < 1 \)

\[ S(0) = 80000, I(0) = 60000, Q(0) = 40000, R(0) = 20000, B(0) = 200000, \]
\[ \mu_1 = 9.13 \times 10^{-5} \text{ / day}, \beta_1 = 0.00025 \text{ / day}, \alpha_1 = 5 \text{ days}, \beta_2 = 0.00015 \text{ / day}, \]
\[ \alpha_2 = 10 \text{ days}, d_1 = 0.015 \text{ / day}, \alpha = 0.2 \text{ / day}, \delta = 0.005 \text{ / day}, \epsilon = 0.2 \text{ / day}, \]
\[ d_2 = 0.0001 \text{ / day}, \eta = 10 \text{ cells / litre / day / person}, \mu_2 = 0.33 \text{ / day}, \omega = 0.2 \text{ / day}, \]
\[ \theta = 0.4, R_q = 0.018 < 1. \]
Figure 2 SIQS cholera transmission model when $R_0 < 1$.
Case II: Disease persists at $R_q > 1$

$S(0) = 80000$, $I(0) = 20000$, $Q(0) = 40000$, $R(0) = 50000$, $B(0) = 200000$,
$\mu_1 = 9.13 \times 10^{-5}$ / day, $\beta_1 = 0.25$ / day, $\alpha_1 = 5$ days, $\beta_2 = 0.0015$ / day,
$\alpha_2 = 10$ days, $d_1 = 0.002$ / day, $\alpha = 0.00002$ / day, $\delta = 0.5$ / day, $\varepsilon = 0.2$ / day,
$d_2 = 0.0001$ / day, $\eta = 10$ cells / litre / day / person, $\mu_2 = 0.33$ / day, $\omega = 0.9$ / day,
$\theta = 0.8$, $R_q = 5.404 > 1$.

![Figure 3 SIQS cholera transmission model when $R_q > 1$]

7. Conclusion
In this paper, we have discussed asymptotic behavior for an SIQS cholera epidemic model mathematically and numerically. We have obtained disease-free and endemic equilibria for the model and analyzed the stability criteria for both equilibria. Mathematically, we have concluded that if basic reproduction number $R_q < 1$, then the disease-free equilibrium is local and global asymptotically stable using Routh-Hurwitz criteria and Castillo-Chavez criteria respectively. If basic reproduction number $R_q > 1$, then the endemic equilibrium is local and
global asymptotic stable using Routh-Hurwitz criteria and Dulac’s plus Poincare criteria respectively. Numerically calculations have been done. Plot graphs of $SIQRB$ vs time $t$ which give more clarity when the disease dies out and when the disease persists.

References