AMGL CODING TECHNIQUES WITH SKOLEM MEAN LIKE LABELING AND TWO STAR GRAPHS

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Abstract

In this paper, a method of coding technique is developed using graph labeling and AMGL code. There are two illustrations for coding a text messages using two star graphs \( K_{1,\lambda} \wedge K_{1,\mu} \) by applying skolem Mean like labeling on it for two different cases \( \mu = \lambda + 1 \) and \( \mu = \lambda + 4 \).

Keywords: AMGL, Star, Skolem Mean like graph, Alphabets maneuvered.
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1. Introduction

In [2][11][12], Uma Maheswari et al. have introduced GMJ coding through a two star and three star with super mean labeling and Fibonacci webs with difference cordial labeling. In [2][3][8], they done a coding techniques with mean labeling and even with difference cordial labeling. In [7], they have introduced AMGL coding through a caterpillar graph with felicitous labeling. In [5] they proved that two star \( K_{1,\lambda} \wedge K_{1,\mu} \) is a skolem mean like graph if and only if \( |\lambda - \mu| \leq 4 \). By referring the above results we got motivated and found a few techniques of coding by applying skolem mean like labeling on two star graph.

2. DEFINITIONS

Definition 2.1 Skolem mean like labeling A graph \( G = (V,E) \) with \( p \) nodes and \( q \) links is said to be a skolem mean like graph if there exists a function \( \psi \) from the node set of \( G \) to \( \{1,2,...,p\} \) such that the induced map \( \psi^* \) is defined by

\[
\psi^*(e = gh) = \begin{cases} 
\frac{\psi(g) + \psi(h)}{2} & \text{if } \psi(g) + \psi(h) \text{ is even} \\
\frac{\psi(g) + \psi(h) + 1}{2} & \text{if } \psi(g) + \psi(h) \text{ is odd}
\end{cases}
\]

then the resulting links get unique labels \( \{2,3,...,p\} \).

Definition 2.2 Wedge

A Wedge is a link which is used for connecting two components of a graph. It is denoted as \( \wedge \), \( \omega(G) < \omega(G) \) where \( \omega \) denotes the number of components of the graph.

AMGL coding method:

A Coding technique is developed by a combination of alphabets maneuvered and a graph which is labeled. This technique is called Alphabets maneuvered graph labeled coding technique (AMGL). AMGL also stands for Antony Maria Gabriel, the father of one of the Researchers Gabriel Margaret Joan.

3. Descriptions for Graph Labeling

By referring the theorem, The two star \( K_{1,\lambda} \wedge K_{1,\mu} \) is a skolem mean like graph if and only if \( |\lambda - \mu| \leq 4 \). Let \( \lambda \leq \mu \) There are five cases viz.

\( \mu = \lambda, \mu = \lambda + 1, \mu = \lambda + 2, \mu = \lambda + 3 \) and \( \mu = \lambda + 4 \).

Let us consider the case \( \mu = \lambda + 1 \). Consider the graph \( G = K_{1,\lambda} \wedge K_{1,\mu} = K_{1,\lambda} \wedge K_{1,\lambda + 1} \). Let \( \{\alpha\} \cup \{\alpha_u : 1 \leq u \leq \lambda\} \) be the nodes of \( K_{1,\lambda} \) and \( \{\beta\} \cup \{\beta_v : 1 \leq v \leq \lambda + 1\} \) be those \( K_{1,\lambda + 1} \).

Then \( G \) has \( 2\lambda + 3 \) nodes and \( 2\lambda + 2 \) links. We have \( V(G) = \{\alpha,\beta\} \cup \{\alpha_u : 1 \leq u \leq \lambda\} \cup \{\beta_v : 1 \leq v \leq \lambda + 1\} \) the appropriate node labeling.

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The respective link labels are given below:

The link label of \( \alpha \alpha \) is \( u + 1 \) for \( 1 \leq u \leq \lambda \) and \( \beta \beta \) is \( \lambda + \nu + 2 \) for \( 1 \leq \nu \leq \lambda - 1 \).

Also, the link label of \( \alpha \beta \) is \( \lambda + 2 \) and \( \beta \beta + 1 \) is \( 2 \lambda + 3 \). Therefore, the respective link labels of

\[ G = \{ 2, 3, \ldots, \lambda + 1, \lambda + 2, \lambda + 3, \ldots, 2 \lambda + 2, 2 \lambda + 3 \} \]

Therefore, the number of unique link label is \( ((2 \lambda + 3) - 2 + 1)) = 2 \lambda + 2 \) unique links.

Consider the Case \( \mu = \lambda + 4 \).

Consider the graph \( G = K_1, \lambda \land K_1, \mu = K_1, \lambda \land K_1, \lambda + 4 \).

Let \( \{ \alpha \} \cup \{ \alpha_u : 1 \leq u \leq \lambda \} \) be the nodes of \( K_1, \lambda \land K_1, \lambda + 4 \).

Then \( G \) has \( 2 \lambda + 6 \) nodes and \( 2 \lambda + 5 \) links.

We have \( V(G) = \{ \alpha, \beta \} \cup \{ \alpha_u : 1 \leq u \leq \lambda \} \cup \{ \beta_v : 1 \leq v \leq \lambda + 4 \} \)

The appropriate node labeling \( \psi : V(G) \rightarrow \{ 1, 2, \ldots, 2 \lambda + 3 \} \) is defined as follows:

\[
\begin{align*}
\psi(\alpha) &= 2 \\
\psi(\beta) &= 2 \lambda + 2 \\
\psi(\alpha_u) &= 2u - 1 \text{ for } 1 \leq u \leq \lambda \\
\psi(\beta_v) &= 2v + 2 \text{ for } 1 \leq v \leq \lambda + 1 \\
\psi(\beta_{\lambda+1}) &= 2 \lambda + 3 \\
\psi(\beta_{\lambda+2}) &= 2 \lambda + 4 \\
\psi(\beta_{\lambda+4}) &= 2 \lambda + 6.
\end{align*}
\]

The respective link labels are given below:

The link label of \( \alpha \alpha \) is \( u + 2 \) for \( 1 \leq u \leq \lambda \) and \( \beta \beta \) is \( \lambda + \nu + 2 \) for \( 1 \leq \nu \leq \lambda + 2 \).

Also, the link label of \( \alpha \beta \) is \( 2 \lambda + 5 \) and \( \beta \beta + 4 \) is \( 2 \lambda + 6 \). Therefore, the respective link labels of \( G = \{ 2, 3, \ldots, \lambda + 1, \lambda + 2, \lambda + 3, \ldots, 2 \lambda + 4, 2 \lambda + 5, 2 \lambda + 6 \} \)

Therefore the number of unique link label is \( ((2 \lambda + 6) - 2 + 1)) = 2 \lambda + 5 \) unique links.

4. Descriptions for AMGL coding Technique

The below flowchart explains the description for AMGL coding technique.
Alphabets maneuvered

<table>
<thead>
<tr>
<th>1</th>
<th>11</th>
<th>13</th>
<th>5</th>
<th>6</th>
<th>24</th>
<th>15</th>
<th>26</th>
<th>19</th>
</tr>
</thead>
<tbody>
<tr>
<td>A</td>
<td>B</td>
<td>C</td>
<td>D</td>
<td>E</td>
<td>F</td>
<td>G</td>
<td>H</td>
<td>I</td>
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<tr>
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<td>14</td>
<td>12</td>
<td>9</td>
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<td>8</td>
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<td>2</td>
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</tr>
<tr>
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<td>K</td>
<td>L</td>
<td>M</td>
<td>N</td>
<td>O</td>
<td>P</td>
<td>Q</td>
<td>R</td>
</tr>
<tr>
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<td>3</td>
<td>18</td>
<td>22</td>
<td>25</td>
<td>21</td>
<td>7</td>
<td></td>
</tr>
<tr>
<td>S</td>
<td>T</td>
<td>U</td>
<td>V</td>
<td>W</td>
<td>X</td>
<td>Y</td>
<td>Z</td>
<td></td>
</tr>
</tbody>
</table>

Graph

![Figure 2](image)

Figure 2. $K_{1,12} \otimes K_{1,13}$

According to rule (7), the clue for two star graph is (12,13).

Notations

Here $S_1$ and $S_2$ are used to refer the first and second star respectively. $C$ and $R_i$ denote the center vertex and the $i^{th}$ ray respectively.

Cipher Text

$J$ takes the value 4. 4 is allotted to the first ray of second star. Therefore $J$ is denoted as $S_2R_1$. Similarly for $O$. $O$ takes the value 8. 8 is allotted to third ray of second star. so 0 is denoted as $S_2R_3$. Therefore the cipher text for whole message is as below

$S_2R_1S_2R_3S_1R_1S_1R_2S_2R_2$ $S_2R_1S_1R_1S_1R_2S_2R_2S_2R_2$ $S_1R_1S_1R_1S_1R_2S_2R_2$ $S_2R_2S_2R_2S_2R_2S_2R_2S_2R_2$ $S_2R_2S_2R_2S_2R_2S_2R_2S_2R_2$ $S_2R_2S_2R_2S_2R_2S_2R_2S_2R_2$ $S_2R_2S_2R_2S_2R_2S_2R_2S_2R_2$ $S_2R_2S_2R_2S_2R_2S_2R_2S_2R_2$ $S_2R_2S_2R_2S_2R_2S_2R_2S_2R_2$

Illustration 2:

The plain text, clue for the labeling and clue for the graph are the same as in Illustration 1.

The names of fruits are taken for AMGL.

<table>
<thead>
<tr>
<th></th>
<th>Fig</th>
<th>Kiwi</th>
<th>Pear</th>
<th>Yuzu</th>
<th>Dates</th>
</tr>
</thead>
<tbody>
<tr>
<td>6_3</td>
<td>11_4</td>
<td>16_4</td>
<td>25_4</td>
<td>4_5</td>
<td></td>
</tr>
<tr>
<td>Xigua</td>
<td>Cherry</td>
<td>Jujube</td>
<td>Quince</td>
<td>Melinjo</td>
<td></td>
</tr>
<tr>
<td>24_5</td>
<td>3_6</td>
<td>10_6</td>
<td>17_6</td>
<td>13_7</td>
<td></td>
</tr>
<tr>
<td>Avacado</td>
<td></td>
<td></td>
<td></td>
<td></td>
<td></td>
</tr>
</tbody>
</table>

According to rule (5), (FT, 6_3, 11_4, 16_4, 25_4, 4_5, 24_5, 3_6, 10_6, 17_6, 13_7, 1_7) is the required clue.

Alphabets maneuvered

<table>
<thead>
<tr>
<th>8</th>
<th>20</th>
<th>17</th>
<th>13</th>
<th>7</th>
<th>1</th>
<th>3</th>
<th>18</th>
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</tr>
</thead>
<tbody>
<tr>
<td>A</td>
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<td>W</td>
<td>X</td>
<td>Y</td>
<td>Z</td>
<td></td>
</tr>
</tbody>
</table>

Graph

![Figure 3](image)

Figure 3. $K_{1,11} \otimes K_{1,15}$

According to rule (7), the clue for two star graph is (11,15).

Notations

Here $FS$ and $SS$ are used to refer the first and second star respectively. $CN$, $iN$ and $il$ denotes the center node, the $i^{th}$ node and $i^{th}$ link respectively.

Cipher Text

$J$ takes the value 19. 19 is allotted to the Sixth link of second star. Therefore $J$ is denoted as $SS6L$. The cipher text for whole message is as follows: $SS6LS12LF5CNSS9L$, $FS3LFS5CNSS1LSS5L$, $SS10LFSS5L$, $FSCNSS9L$, $SS6LFSS9LFSS5CNSS1LFS5LFSS7L$, $SS1LS512L$, $FS2LSS9LSS12LF5S3L$, $SS1LS5LFS5L$, $SS2LFS5LSS4LF5S7LSS5LSS1L$.  

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6. Conclusion

We investigate skolem Mean like labeling on the two star graph for communicating some messages using alphabets maneuvered and two methods for letter codings are given in this paper. In Future we planned to develop AMGL code on two star graph with Mean labeling for other two cases.

ACKNOWLEDGEMENT

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References

[10] G. Uma Maheswari, G. Margaret Joan Jebarani and V. Balaji GMJ Coding Through a Three Star and Sup-