Developing students' mathematical communication skill in junior high school with various level of mathematics achievement through generative learning model

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Abstract.

Mathematical communication skill was the missing piece of mathematics education failure puzzle for centuries despite its central role in learning activities. Built on top of other process skills, it was the tool to build more comprehensive conceptual understanding by learning math from different perspectives, sharpens other cognitive skills, and provides important feedback about students' understanding. This study observed how the students' mathematical communication skill was correlated with the application of generative learning model and how this correlation was influenced by various level of school's academic achievement in mathematics. Using fractional 2x2-factorial design, careful non-probability sampling combined with simple random sampling to pick 171 students, and a validated and reliable scoring system to measure communication skill, this study proved that the students' mathematical communication skill has a significant correlation with the application of generative learning model. The use of factorial design revealed that this correlation was more determined by the application of generative learning model, not by the level of schools' or students' mathematics achievement. Teachers should be more optimistic about the use of generative learning models in improving mathematical communication skill even in classes with lower level of mathematics achievement.
Keywords: mathematical communication skill, generative learning model, factorial experiment

Introduction

Mathematics education continues to shift from procedure-focused into more conceptual-understanding learning. Students are being facilitated to recall prior knowledge and connect it with new knowledge so they will realize what's new to them and organize this new concept into their memory. This way, students will think further and deeper than previous lesson – lengthen and strengthen their chains of knowledge. This process of generative learning involves five process skills that amplify each other: reasoning and proof, connections, representations, problem solving, and communication. The two last skills were built on top of other skills and reflect a more comprehensive ability to incorporate the new concept being learnt.

Mathematical communication was an intricate yet simplest way to compass the mathematics meaningful learning. As illustrated by Wittrock, the father of generative learning theory, "although a student may not understand sentences spoken to him by his teacher, it is highly likely that a student understands sentences that he generates himself".[1] By communicating and sharing their ideas, students will learn to be clear, convincing, and precise in their use of mathematical language. By listening to others' explanations, students gain opportunities to develop their own understanding in different perspective. This multiple perspective helps students to sharpen their thinking and make connections. To be effective, teacher must manage this communication process in order to prevent students from performing meaningless discussion. Students shall be directed to explain their arguments and rationales, not just the procedures or summaries. To do this, teacher will depend on students' mathematical communication skill to clarify students' understanding, detect misconception or ambiguities, conduct refinement or amendment, or just give some friendly response to engage students deeper in discussion. Mathematical communication provides valuable feedback mechanism for the teacher to navigate the process of generative learning.

Conversely, students' mathematical communication skill itself was proved to be corrigible by the application of generative learning models.[2,3] Despite the increasing trend of researches, the underlying mechanism on this particular interaction yet to be clear. As described previously, the mathematical communication is an indispensable part of the generative learning itself. In many instruction designs, it builds the whole process of generative learning. By practicing the mathematical communication during a generative learning session, students can sharpen their communication skill, understand more, and will be proficient in the next learning sessions. Assuming this mutual effect, perhaps the main concern is not the underlying mechanism behind it, but which factors may inhibit or compound the effect.

The intentional learning science (especially the instructional science) was a model of complexity theory where the unlimited variation in instructional design and its application met the complicated nature of human knowledge system including mathematics. This produces double uncertainties.[4] Aside from being a practical cousin of constructivism, the generative learning theory was also provides a more complete perspective about learning activity, making it a second cousin to behaviorism. The theory brings together our understanding of learning processes and the design of external stimuli or instruction.[5] Stimuli, motivation, attention, self-regulation, and other behavioral processes brings more complexity in generative learning, thus double the uncertainty of its outcome. But, there must be an optimum degree of generality, in certain abstraction, that we can observe, analyze, and conclude.

With an infinite number of factors involved in the improvement of students' mathematical communication skill through generative learning model, the need to use fractional factorial design trials emerges. According to Wikipedia in March 2019, a factorial experiment is an experiment whose design consists of two or more factors, each with discrete possible values or "levels", and whose experimental units take on all possible combinations of these levels across all such factors. Such an experiment allows the investigator to study the effect of each factor on the response variable, as well as the effects of interactions between factors on the response variable. In fractional factorial design, only an adequately chosen fraction of the treatment combinations required for the complete factorial experiment is selected to be run. The decision about which run to make and which to leave out is the subject of interest here.

Wu and Hamada proposed three principles for selecting which fraction of treatment should be run first. For an interaction to be significant, at least one of its parent factors should be significant first (heredity-of-effect principle). These parent factors produce lower-order effects that more important than higher-order effects (hierarchy-of-effect principle) and the number of the relatively important effects is relatively small (sparsity-of-effect principle).[6] Thus, to study the effect of generative learning model in improving students' mathematical communication skill, we must assure the significance of some basic factors and the significance of their interactions before studying their sub factors and interactions. Branch already identified eight entities that are always present in instructional design: student, content, media, teacher, peers, time, goal, and context.[7] We can simplify these entities into two lower-order factors: student and school, while the school factor is the sum of the last seven entities including curriculum, teachers' professionalism, scheduling, day-to-day class management, facilities, and the class social environment.

The purpose of the current study was to observe the effect of generative learning model in improving students' mathematical communication skill in interaction with student factor and school factor as described above. Of course the use of factorial design, especially to study such basic interactions, would results in aliasing of effects and difficulties in generalization. Therefore, the main idea behind this study was to provide supporting facts required to establish hypotheses and designs for any future observations of more detailed sub factors and interactions in generative learning model and students' mathematical communication skill.

2. Methods

2.1. Design

This study was a class experiment using fractional factorial design to observe how the generative learning model, in interaction with student-factor and school-factor, could improve the students' mathematical communication skill. The student-factor was measured by the students' mathematical achievement (score) in the last end-of-grade exam, while the school-factor was the average of graduated students' NEM (*Nilai Ebtanas Murni* or national exit-exam score) for mathematics at the last exit-exam. The population of this study was students in all junior high schools in Gorontalo province of Indonesia. This study used pretest and posttest score to measure the improvement of students' mathematical communication skill after treatment.

2.2. Sampling

To clearly bring forward the effect of difference in schools' and students' mathematics achievement, a non-probability sampling must be conducted at the first stage of sampling to select two schools by their mathematics average NEM. Two schools was picked (SMPN 2 Gorontalo and SMPN Kabila; abbreviated as SMPN2G and SMPNK) for their highest and lowest achievement, respectively, as the source of four 8^{th}-grade classes (four treatment groups) in this 2x2-factorial trial. Before selecting classes, all students' previous (7^{th}) end-of-grade mathematics score were collected from both schools. Normality test and Levene test showed the equality of variance on the normally distributed score – taken independently for each school. After averaging the scores by class, the mean difference test for each school also showed no significant difference, both between classes and within classes. The average score for SMPN2G and SMPNK were 6.9253 and 6.5863, respectively. Based on these results, a simple random sampling was conducted to pick two classes (HG and HC) from the highly-achieving SMPN2G. HG was the experimental class taught with generative learning model while HC was the control class taught with a non-generative approach. The same technique also used to select two classes (LG and LC) from the lowly-achieving SMPNK. From the total number of 171 students in the four classes, a simple random sampling was used to pick 30 students from each class to be sub-grouped by their mathematics achievement into "high" and "low" level of mathematics achievement, 15 students each. Despite of this sub grouping, all 171 students still received the treatment.

2.3. Instruments

This study used two main instruments: the lesson plans and a scoring system developed to measure the mathematical communication skill. Other instruments are students' worksheet, questionnaire sheets, and questionnaire manuals.

Two set of lesson plans developed for this trial were consistent with the mathematics curriculum for 8^{th}-grade students in Indonesia. Both set have similar content and what materials being used, but they were different in learning activities. They consisted of four identical subjects: (1) function and relation, (2) Pythagoras theorem, (3) parallelogram, rhombus, kite, trapezoid, and (4) ratio and proportionality. The first set was for the experimental classes. Therefore, it was fully optimized for generative learning model. The second set for the control classes was developed to minimize the use of generative learning processes.

To measure the mathematical communication skill, this study used a custom scoring system developed by the author. The score was calculated using a questionnaire of 10 items reflecting the degree of quality (correctness) and quantity (comprehensiveness) of students' communication about their mathematical ideas. The scoring system only evaluated the content, not the psychosocial aspects of communication skill. As each item has score ranged from 0 to 5, the maximum total score was 50. The proposed version of this questionnaire consisted of 15 items, but had been truncated after being validated using Pearson product-moment correlation test. The reliability value was 0.71 (or classified as "high" according to Guildford coefficient of reliability).[8] The validation and reliability tests were conducted using Anates® v4.09 (software developed by Karno To and Yudi Wibisono). Earlier, the questionnaire also reviewed and accepted by a team of validator involving mathematics experts and teachers. The questionnaire was filled by the teachers who lead the class and the score was calculated by the researcher. The test was conducted twice as pretest and posttest.

2.4. Experiment procedure

Earlier before treatment, a pilot study was conducted in one of 8th-grade class in SMPN 1 Gorontalo in order to: (1) test and refine the content and tool to be used in real experiment, (2) train and select the eligible teachers and observers, (3) study the teachers' problems while delivering the lesson and adjust the class procedure if necessary, and (4) study the students' cognitive and social response including how the mathematical communication skill could be improved, then make some adjustment on the lesson plan or other aspects of the research.

Before starting the treatment, a pretest of mathematical communication skill was carried on the last mathematics session at each class, just before the next mathematics session involving the treatment agenda. The real treatment consisted of 16 lesson sessions. Four sessions for each class were conducted in different days. Generative learning model in the experimental classes (HG and LG) was conducted with five steps of activities through full discussion: (1) orientation, (2) idea mining and presentation, (3) challenge and reconstruction, (4) application, and (5) recheck and refine. The teachers were responsible to guide the process of problem solving by recalling and applying the previously acquired mathematical concepts. In the closing phase, the students were asked to recognize the new concepts they already learnt and to share their ideas about how to deal with other similar problems. Meanwhile, the learning model for the control classes (HC and LC) was consisted of slide presentation by teacher followed by questioning and discussion phase. The role of the teacher here was just to deliver the concept, ensure that the students understand by asking them, and answer any question. Any generative learning processes occurred in the control classes were occurred naturally without being forced intentionally through scaffolding as done in the experimental classes.

The mathematical communication skill measurement was conducted in each class during the last session. A cross-observation between teachers and students was also conducted via additional questionnaires to provide complementary data about teachers' and students' performance, problems, and suggestions during the treatment sessions.

2.5. Data analyses

This study was to observe the effect of an independent variable (the application of generative learning model as categorical variable) and two other independent variables (the school's mathematics achievement and the student's mathematics achievement – both are simplified as categorical variables) on a single dependent variable (the improvement in mathematical communication skill as continuous numeric or interval variable). To evaluate this effect within the context of its interaction with various combination of schools' and students' mathematics achievement, the sample was divided into four analysis groups: highly-achieving students in highly-achieving school (St_HSc_H), lowly-achieving students in highly-achieving school (St_LSc_H), highly-achieving students in lowly-achieving school (St_HSc_L), and lowly-achieving students in lowly-achieving school (St_LSc_L).

As the dependent variable was proved later to be normally distributed ($\alpha = 0.05$), then the most suitable parametric analysis to test the inter-correlation was the factorial ANOVA. Four ANOVA tests were also done to evaluate the effect on the four analysis groups. All calculation was done using SPSS for Windows version 11.5.

3. Results

3.1. Gain scores of the mathematical communication skill

Table 1 summarizes the pretest and posttest mean score of students' mathematical communication skill for each group and the improvements after treatment. Groups with generative learning model got the most improvement in their mathematical communication skill after the treatment, especially in groups with higher mathematics achievement. The generative learning model improves the mathematical communication skill of the highly-achieving students in highly-achieving school (St_HSc_H group) up to 18.73 (±1.73) pts (or 152.65% of their pretest score) compared with the lowly-achieving student in lowly-achieving school (St_LSc_L group) which only increased 5.66 (±2.19) pts (or 48.5% of the pretest score).

Table 1. Pretest mean score, posttest mean score, and the mean gain percentage of students' mathematical communication ability by groups.

Generative learning	School level	Student level	N	Pre test mean score	Post test mean score	Improvement (gain score) in mathematics communication skill					
						Mean	Gain %	Mean	Gain %	Mean	Gain %
Yes	High (HG)	High	15	12.27	31.00	18.73	152.65	15.54	128.39	13.97	115.43
		Low	15	11.93	24.27	12.34	103.44				
	Low (LG)	High	15	12.53	26.20	13.67	109.10	12.40	102.48		
		Low	15	11.67	22.80	11.13	95.37				
No	High (HC)	High	15	12.13	23.47	11.34	93.49	8.57	72.23	8.47	71.76
		Low	15	11.60	17.40	5.80	50.00				
	Low (LC)	High	15	11.80	22.87	11.07	93.81	8.37	71.28		
		Low	15	11.67	17.33	5.66	48.50				

By design, the purpose of this study was to observe the improvement of mathematical communication skill by comparing the posttest and pretest score, in other words, by evaluating the gain score. But there was a common pitfall in computing the difference between pre- and posttest score. The gain score controls for individual differences in pretest scores by measuring the posttest score relative to the each student's pretest score. But, the gain score analysis does not control for the differences in pretest scores between the two groups collectively. As happened in this study, there was an enormity of gain score in the highly-achieving groups that rose twice beyond expectation that might reflect students' difference in adapting the pretest and posttest technical aspects (or other interacting factors) that cannot be easily avoided. Therefore, to minimize bias, this study used the posttest mean score instead to observe the mathematical communication skill improvement for the next analyses. The posttest score itself was reliable enough to reflect the latest students' mathematical communication skill because the difference of effect of various input from the three independent factors still taken into account.

3.2. The effect of generative learning model on students' posttest mean score

As the Kolmogorov-Smirnov test yielded a normally distributed posttest mean scores and the Levene test to assess the equality of variance also displayed a homogenous results across all analysis groups, four tests of ANOVA were used to analyze the effect on those groups (**Table 2**). The data showed a significant correlation in all but the last group (St_LSc_L). This result suggest that a generative learning model could improve students' mathematical communication skill in almost all levels of schools' or students' mathematics achievement, except for the lowly-achieving students in lowly-achieving schools.

Table 2. ANOVA test results on four analysis groups

Analysis group		Sum of Squares	df	Mean Square	F	Sig.
St_HSc_H	Between Groups	2,745.633	1	2,745.633	53.586	.000
	Within Groups	1,434.667	28	51.238		
	Total	4180.300	29			
St_LSc_H	Between Groups	681.633	1	681.633	13.068	.001
	Within Groups	1,460.533	28	52.162		
	Total	2,142.167	29			
St_HSc_L	Between Groups	672.133	1	672.133	13.228	.001

	Within Groups	142.667	28	50.810		
	Total	2,094.800	29			
St_LSc_L	Between Groups	145.200	1	145.200	2.801	.105
	Within Groups	1,451.600	28	51.843		
	Total	1,596.800	29			

3.3. The factorial effects and interactions on students' posttest mean score

The factorial ANOVA result displayed significant correlation of the generative learning model, students' level, and schools' level with the students' posttest mean score (p = 0.05) (**Table 3**). The same result was also found in interaction models involving the application of generative learning model. However, there was no significant correlation in interaction models involving both the school's level and student's level.

Table 3. The factorial ANOVA test result

No.	Source	Sum of Squares	df	Mean Square	F	Sig.
1.	**Independent variables**					
	The application of generative learning model	5,757.356	1	5,757.356	111.678	.000
	School's level	683.511	2	341.756	6.629	.002
	Student's level	1,155.200	1	1,155.200	22.408	.000
2.	**Interactions**					
	The application of generative learning model and school's level	452.044	2	226.022	4.384	.014
	The application of generative learning model and student's level	417.089	1	417.089	8.090	.005
	School's level and student's level	64.133	2	32.067	.622	.538
	All variables	73.378	2	36.689	.712	.492
3.	**Within groups**	8,660.933	168	51.553		
	Total	17,263.644	179			

4. Discussion

The use of fractional factorial design in social studies such as learning and teaching researches was very helpful. It was almost impossible to observe the relationship between variables while there are so many other factors must be taken into account. If the number of very dominant factors is small or there is only a single dominant factor, the best solution is to focus on those very dominant factors and neglect other insignificant factors; in other words, study the most significant fraction of the full factorial design.

In the current study, the effect of generative learning model in improving students' mathematical communication skill was evaluated along with two important, lower-order factors in the process of instruction, i.e. the schools' and students' academic achievement in mathematics, to provide a more comprehensive analysis on mathematics education dynamics. The use of four models of interaction in this study was intended to examine how the "school-factor" and "student-factor" might interact each other and influence the improvement of mathematical communication skill. The school factor was the sum of school's infrastructures, teachers' professionalism, an engaging learning community, quality assurance, or simply the day-to-day management of class routines. This quality could give a strong influence on the student factor such as their opportunity to learn effectively and improve their ability.

The first interaction model (the highly-achieving students in highly-achieving schools) was the situation in which the talented students found their suitable environment to optimize their ability. On the other hand, the last interaction model (the lowly-achieving students in lowly-achieving schools) was the situation in which the low quality of student's achievement might be influenced by the low quality of their school itself. The two other interaction models might reflect the situation in which the quality of school failed to transfer significant influence on student's quality (to be better or worse). This study proved the effect of the last interaction model (low quality students in low quality school) in limiting students' improvement in mathematical communication skill, despite the significant effect of generative learning model in improving the skill on the other interaction models.

However, this study also suggests that the effect of generative learning model was significant enough to improve mathematical communication skill in lowly-achieving school or for lowly-achieving students. Because the skill itself could provide a boost for the student to improve their conceptual understanding in mathematics, therefore, the application of generative learning model could accelerate the improvement in mathematics education in short or long term, despite the schools' level of mathematics achievement.

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