A NEW CLOSURE OPERATOR IN INTUITIONISTIC TOPOLOGICAL SPACES

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Abstract- The aim of this paper is to introduce a new concept of intuitionistic generalized closure operator and intuitionistic generalized interior operator namely *Icl** and *Iint** respectively. Also we study some of their properties and the realtion between *Icl** and *Iint**.

Index Terms*- Icl*, Iint*, Ig – closed set, Ig – open set*

I. Introduction

The concept of intuitionistic set was introduced by D.Coker [2] in 1996, and also he [1] has introduced the concept of intuitionistic topological space. Dunham [5] introduced the concept of generalized closure operator c^* using generalized closed sets of Levine [6]. Then Younis J. Yaseen and Asmaa G. Raouf [8] introduced the concept of intuitionistic generalized closed sets. In this article, we define a new operator namely intuitionistic generalized closure operator in intuitionistic topological space and discussed their properties. Also we proved intuitionistic closure operator is a Kuratowski closure operator and their properties are determined.

2. PRELIMINARIES

In this section, we list some definition and fundamental results which are to be used further.

Definition 2.1 [1] Let X be a nonempty fixed set. An *intuitionistic set* (IS in short) $\tilde{A}$ is an object having the form $\tilde{A} =< X, A^1, A^2 >$ where A^1 and A^2 are subsets of X such that $A^1 \cap A^2 = \emptyset$. The set A^1 is called the set of *member* of $\tilde{A}$, while A^2 is called the set of *non member* of $\tilde{A}$.

Definition 2.2 [1] Let X be a non empty set, $\tilde{A} =< X, A^1, A^2 >$ and $\tilde{B} =< X, B^1, B^2 >$ be an IS's and let $\{ \tilde{A}_i : i \in J\}$ be arbitrary family of IS's, where $\tilde{A}_i =< X, A_i^1, A_i^2 >$. Then the following hold.

(i) $\tilde{A} \subseteq \tilde{B}$ if and only if $A^1 \subseteq B^1$ and $B^2 \subseteq A^1$.
(ii) $\tilde{A} = \tilde{B}$ if and only if $\tilde{A} \subseteq \tilde{B}$ and $\tilde{B} \subseteq \tilde{A}$.
(iii) $\overline{A} =< X, A^2, A^1 >$ is called the complement of $\tilde{A}$. It is aslo denoted by $X – \tilde{A}$.
(iv) $\cup \tilde{A}_i =< X, \cup A_i^1 ,\cap A_i^2 >$.
(v) $\cap \tilde{A}_i =< X, \cap A_i^1 ,\cup A_i^2 >$.
(vi) $\tilde{A} - \tilde{B} = \tilde{A} \cap \tilde{B}^c$.
(vii) $\tilde{\emptyset} =< X, \emptyset, X >$ and $\tilde{X} =< X, X, \emptyset >$.

Definition 2.3 [1] Let X be a nonempty set and τ be the family of IS's of X. Then τ is called an *intuitionistic topology* (IT in short) on X if it satisfies the following axioms.
(a) $\tilde{\emptyset}$, $\tilde{X} \in \tau$
(b) $\tilde{G}_1 \cap \tilde{G}_2 \in \tau$ for every $\tilde{G}_1$, $\tilde{G}_2 \in \tau$
(c) $\cup \tilde{G}_i \in \tau$ for any arbitrary family $\{\tilde{G} : i \in J\} \subseteq \tau$.

The pair (X, τ) is called an *intuitionistic topological space* (ITS in short) and any IS G in τ is called an intuitionistic *open set* (IOS). The complement $\overline{A}$ of an IO set $\tilde{A}$ in an ITS (X, τ) is called an *intuitionistic closed set* (ICS).

Definition 2.4 [1] Let (X, τ) be an ITS and $\tilde{A} =< X, A^1, A^2 >$ be an IS in X. Then the interior and the closure of A are denoted by $Iint(\tilde{A})$ and $Icl(\tilde{A})$, and are defined as follows.
$Iint(\tilde{A}) = \cup \{\tilde{G} \mid \tilde{G}$ is an IOS and $\tilde{G} \subseteq \tilde{A}\}$ and
$Icl(\tilde{A}) = \cap \{\tilde{K} \mid \tilde{K}$ is an ICS and $\tilde{A} \subseteq \tilde{K}\}$.

Definition 2.5 [2] Let X be a nonempty set and $p \in X$ be a fixed element. Then the IS $\tilde{p}$ defined by $\tilde{p} =< X, \{p\}, \{p\}^c >$ is called an *intuitionistic point* (in short, IP).

Definition 2.6 [8] Let (X, τ) be an ITS and $\tilde{A} =< X, A^1, A^2 >$ be an IS in X, $\tilde{A}$ is said to be *intuitionistic generalized closed set* (briefly *Ig – closed set*) $Icl(\tilde{A}) \subseteq \tilde{U}$ whenever $\tilde{A} \subseteq \tilde{U}$ and $\tilde{U}$ is IO in X.

Definition 2.7 [8] Let (X, τ) be an ITS and $\tilde{A} =< X, A^1, A^2 >$ be an IS in X, $\tilde{A}$ is said to be *intuitionistic generalized open set* (briefly *Ig – open set*) if $X – \tilde{A}$ is *Ig – closed* set in X.

Theorem 2.8 [8] Let (X, τ) be an ITS. Then the following properties hold.

- **(i)** Every intuitionistic closed set is *Ig – closed*.
- **(ii)** Union of two *Ig – closed* set is *Ig – closed*.

3. THE INTUITIONISTIC GENERALIZED CLOSURE OPERATOR

Definition 3.1 For an ITS (X, τ), let $\mathscr{D} = \{\tilde{A} : \tilde{A} \subseteq IS(X) and\ \tilde{A}\ is\ Ig - closed\}$.

Definition 3.2 If $\tilde{A}$ is an IS of an ITS (X, τ), then the intuitionistic generalized closure of $\tilde{A}$ is defined as the intersection of all Ig – closed sets in X containing $\tilde{A}$ and is denoted by $Icl^*(\tilde{A})$.

That is, $Icl^*(\tilde{A}) = \cap \{\tilde{E}: \tilde{A} \subseteq \tilde{E} \in \mathscr{D}\}$.

Example 3.3 Let $X = \{i, j, k\}$ and $\tau = \{\tilde{X}_I, \tilde{\emptyset}_I, < X, \{j\},\{i, k\} >, < X, \{i\},\{j\} >, < X, \{i, j\}, \emptyset >\}$ be an ITS. Let $\tilde{A} = < X, \{j\}, \emptyset >$. Then $Icl^*(\tilde{A}) = < X, \{j, k\}, \emptyset >$.

Theorem 3.4 Let (X, τ) be an ITS. If $\tilde{A}$ and $\tilde{B}$ are IS of X, then the following properties hold.

- **(i)** $Icl^*(\tilde{X}_I) = \tilde{X}_I$
- **(ii)** $Icl^*(\tilde{\emptyset}_I) = \tilde{\emptyset}_I$
- **(iii)** $\tilde{A} \subseteq Icl^*(\tilde{A})$
- **(iv)** If $\tilde{B}$ is any Ig - closed set containing $\tilde{A}$, then $Icl^*(\tilde{A}) \subseteq \tilde{B}$.

Proof: Follows from the definition 3.2.

Theorem 3.5 Let (X, τ) be an ITS and $\tilde{A}$ be an IS of X. Then the following properties hold.

- **(i)** $\tilde{A} \subseteq Icl^*(\tilde{A}) \subseteq Icl(\tilde{A})$.
- **(ii)** If $\tilde{A}$ is Ig – closed then $Icl^*(\tilde{A}) = \tilde{A}$.

Proof: (i) $\tilde{A} \subseteq Icl^*(\tilde{A})$ follows from the theorem 3.4 (ii). Suppose that $\tilde{A}$ is intuitionistic closed set. Then $\tilde{A}$ is Ig – closed. So {Intuitionistic closed set containing $\tilde{A}$} $\subseteq$ {Ig – closed set containing $\tilde{A}$}. $\cap$\{ Ig – closed set containing $\tilde{A}$\} $\subseteq \cap$\{ Intuitionistic closed set containing $\tilde{A}$\}. That is, $Icl^*(\tilde{A}) \subseteq Icl(\tilde{A})$.
(ii) Follows from definition 3.2 and theorem 3.5 (i).

Remark 3.6 The containment relations in theorem 3.5 (i) may be strict or equal and the converse of the theorem 3.5 (ii) is not true in general as seen from the succeeding examples.

Example 3.7 Let $X = \{i, j, k\}$ and $\tau = \{\tilde{X}_I, \tilde{\emptyset}_I, < X, \{i\},\{k\} >, < X, \{k\},\{i, j\} >, < X, \{i, k\}, \emptyset >\}$ be an ITS. Let $\tilde{A} = < X, \{i, j\}, \{k\} >$. Then $Icl(\tilde{A}) = < X, \{i, j\}, \{k\} >$ and $Icl^*(\tilde{A}) = < X, \{i, j\}, \{k\}>$. Therefore $\tilde{A} = Icl^*(\tilde{A}) = Icl(\tilde{A})$.

Let $\tilde{A} = < X, \emptyset, \{k\})$. Then $Icl(\tilde{A}) = < X, \{i, j\}, \{k\}>$ and $Icl^*(\tilde{A}) = < X, \{j\}, \{k\}>$. Therefore $\tilde{A} \subset Icl^*(\tilde{A}) \subset Icl(\tilde{A})$.

Example 3.8 Let $X = \{i, j, k\}$ and $\tau = \{\tilde{X}_I, \tilde{\emptyset}_I, < X, \emptyset,\{i, j\}>, < X,\{i\},\emptyset>, < X, \{i\}, \{k\}>\}$ be an ITS. Let $\tilde{A} = < X, \emptyset,\{j\}>$. Then $Icl^*(\tilde{A}) = < X, \emptyset,\{j\}> = \tilde{A}$ but $\tilde{A}$ is not Ig – closed set because $\tilde{A} \subseteq < X,\{i\},\emptyset>$ but $Icl(\tilde{A}) \supseteq < X,\{i\},\emptyset>$.

Theorem 3.9 Let (X, τ) be an ITS and $\tilde{A}$, $\tilde{B}$ be any two IS of X. Then the following results hold.

- **(i)** If $\tilde{A} \subseteq \tilde{B}$, then $Icl^*(\tilde{A}) \subseteq Icl^*(\tilde{B})$.
- **(ii)** $Icl^*(\tilde{A} \cap \tilde{B}) \subseteq Icl^*(\tilde{A}) \cap Icl^*(\tilde{B})$.
- **(iii)** If $\tau_1 \subseteq \tau_2$, then τ_1- $Icl^*(\tilde{A}) \subseteq \tau_2$ - $Icl^*(\tilde{A})$.

Proof: (i) Let $\tilde{A} \subseteq \tilde{B}$. By definition 3.2 $Icl^*(\tilde{B}) = \cap \{\tilde{E}: \tilde{B} \subseteq \tilde{E} \in\mathscr{D}\}$. If $\tilde{B} \subseteq \tilde{E} \in\mathscr{D}$, then $\tilde{A} \subseteq \tilde{B} \subseteq \tilde{E} \in \mathscr{D}$. We have $Icl^*(\tilde{A}) \subseteq \tilde{E}$. Then $Icl^*(\tilde{A}) \subseteq \cap \{\tilde{E}: \tilde{B} \subseteq \tilde{E} \in \mathscr{D}\} = Icl^*(\tilde{B})$. That is $Icl^*(\tilde{A}) \subseteq Icl^*(\tilde{B})$.
(ii) We have $\tilde{A} \cap \tilde{B} \subseteq \tilde{A}$ and $\tilde{A} \cap \tilde{B} \subseteq \tilde{B}$. Then by (i), $Icl^*(\tilde{A} \cap \tilde{B}) \subseteq Icl^*(\tilde{A})$ and $Icl^*(\tilde{A} \cap \tilde{B}) \subseteq Icl^*(\tilde{B})$. Thus $Icl*(\tilde{A} \cap \tilde{B}) \subseteq Icl^*(\tilde{A}) \cap Icl^*(\tilde{B})$.
(iii) Given $\tau_1 \subseteq \tau_2$, which implies that $\mathscr{D}(\tau_2) \subseteq \mathscr{D}(\tau_1)$, implies that $\{\tilde{E}: \tilde{A} \subseteq \tilde{E} \in \mathscr{D}(\tau_2)\} \subseteq \{\tilde{F}: \tilde{A} \subseteq \tilde{F} \in \mathscr{D}(\tau_1)\}$.Then $\cap\{\tilde{F}: \tilde{A} \subseteq \tilde{F} \in \mathscr{D}(\tau_1)\} \subseteq \cap\{\tilde{E}: \tilde{A} \subseteq \tilde{E} \in \mathscr{D}(\tau_2)\}$. Thus τ_1- $Icl^*(\tilde{A}) \subseteq \tau_2$ - $Icl^*(\tilde{A})$.

Theorem 3.10 The operator Icl^* is a Kuratowski closure operator.

Proof: (i) It follows from the theorem 3.4 (ii) that $Icl^*(\tilde{\emptyset}_I) = \tilde{\emptyset}_I$.
(ii) $\tilde{A} \subseteq Icl^*(\tilde{A})$ follows from the theorem 3.5 (i).
(iii) Suppose $\tilde{A}$ and $\tilde{B}$ are two IS of X. Then by theorem 3.9 (i), $Icl^*(\tilde{A}) \subseteq Icl^*(\tilde{A} \cup \tilde{B})$ and $Icl^*(\tilde{B}) \subseteq Icl^*(\tilde{A} \cup \tilde{B})$. Hence we have $Icl^*(\tilde{A}) \cup Icl^*(\tilde{B}) \subseteq Icl^*(\tilde{A} \cup \tilde{B})$. Now if $\tilde{p} \notin Icl^*(\tilde{A}) \cup Icl^*(\tilde{B})$ then there exist $\tilde{E}, \tilde{F} \in \mathscr{D}$ such that $\tilde{A} \subseteq \tilde{E}$, $\tilde{p} \notin \tilde{E}$ and $\tilde{B} \subseteq \tilde{F}, \tilde{p} \notin \tilde{F}$. Hence $\tilde{A} \cup \tilde{B} \subseteq \tilde{E} \cup \tilde{F}$ and $\tilde{p} \notin \tilde{E} \cup \tilde{F}$. Since $\tilde{E} \cup \tilde{F}$ is Ig - closed set, $\tilde{p} \notin Icl^*(\tilde{A} \cup \tilde{B})$. Then we have $Icl^*(\tilde{A} \cup \tilde{B}) \subseteq Icl^*(\tilde{A}) \cup Icl^*(\tilde{B})$. Therefore $Icl^*(\tilde{A} \cup \tilde{B}) = Icl^*(\tilde{A}) \cup Icl^*(\tilde{B})$.
(iv) If $\tilde{A} \subseteq \tilde{E} \in \mathscr{D}$, then $Icl^*(\tilde{A}) \subseteq \tilde{E}$ and $Icl^*(Icl^*(\tilde{A})) \subseteq \tilde{E}$ by definition of Icl^*. Hence $Icl^*(Icl^*(\tilde{A})) \subseteq \cap \{\tilde{E}: \tilde{A} \subseteq \tilde{E} \in \mathscr{D}\} = Icl^*(\tilde{A})$. Conversely $Icl^*(\tilde{A}) \subseteq Icl^*(Icl^*(\tilde{A}))$ is true by theorem 3.5 (i). Thus we have $Icl^*(Icl^*(\tilde{A})) = Icl^*(\tilde{A})$.
By (i) to (iv), the operator Icl^* is the Kuratowski closure operator.

Definition 3.11 If (X, τ) is an ITS, let τ^* be the topology on X defined by the closure operator Icl^*. That is, $\tau^* = \{\tilde{G} \in IS(X) : Icl^*(X - \tilde{G}) = X - \tilde{G}\}$.

Theorem 3.12 Let (X, τ) be an ITS. Then $\tau \subseteq \tau^*$.

Proof: Let $\tilde{G}$ be any intuitionistic open set. It follows that $X - \tilde{G}$ is intuitionistic closed set. Therefore $X - \tilde{G}$ is a Ig –closed set. Hence $Icl^*(X - \tilde{G}) = X - \tilde{G}$, by theorem 3.5 (ii).That is $\tilde{G} \in \tau^*$, and hence $\tau \subseteq \tau^*$.

Remark 3.13 The containment relation in the above theorem 3.12 may be proper as seen from the succeeding example.

Example 3.14 Let $X = \{i, j, k\}$ and $\tau = \{\tilde{X}_I, \tilde{\emptyset}_I, < X, \emptyset,\{j\}>, < X, \{i\},\{j\}>, < X, \{k\},\emptyset>, < X, \{i, k\}, \emptyset>\}$ be an ITS. Then $\tau^* = \{\tilde{X}_I, \tilde{\emptyset}_I, < X, \emptyset,\{j\}>, < X, \emptyset,\{i, j\}>, < X, \emptyset,\{j, k\}>, < X,\{i\},\{j\}>, < X,\{i\},\{j, k\}>, < X, \{k\},\{j\}>, < X,\{k\},\{i, j\}>, < X, \{k\},\emptyset>, < X, \{i, k\},\{j\}>, < X,\{i, k\}, \emptyset>\}$. Clearly $\tau \subset \tau^*$.

Theorem 3.15 Let (X, τ) be an ITS. If an IS $\tilde{A}$ of X is Ig – closed, then $\tilde{A}$ is τ^* – closed.

Proof: Let $\tilde{A}$ be an Ig – closed set. Then by theorem 3.5 (ii), $Icl^*(\tilde{A}) = \tilde{A}$. That is $Icl^*(X – (X – \tilde{A})) = (X – (X – \tilde{A}))$. It follows that $X – \tilde{A} \subseteq \tau^*$. Therefore $\tilde{A}$ is τ^* – closed. ■

Remark 3.16 The converse of the above theorem need not be true as seen from the succeeding example.

Example 3.17 In example 3.8, the IS $\tilde{A} = < X, \emptyset, \{j\} >$ is τ^* – closed but not Ig – closed set in (X, τ).

4. THE INTUITIONISTIC GENERALIZED INTERIOR OPERATOR

Definition 4.1 If $\tilde{A}$ is an IS of an ITS (X, τ), then the intuitionistic generalized interior of $\tilde{A}$ is defined as the union of all Ig – open sets in X that are contained in $\tilde{A}$ and is denoted by $Iint^*(\tilde{A})$. That is, $Iint^*(\tilde{A}) = \cup \{\tilde{E} : \tilde{E}\ is\ Ig - open\ sets\ and\ \tilde{E} \subseteq \tilde{A}\}$.

Example 4.2 Let $X = \{i, j, k\}$ and $\tau = \{\widetilde{X}_I, \widetilde{\emptyset}_I, < X, \{k\}, \{i, j\} >, < X, \{i\}, \{k\} >, < X, \{i, k\}, \emptyset > \}$ be an ITS. Let $\tilde{A} = < X, \emptyset, \{i\} >$. Then $Iint^*(\tilde{A}) = < X, \emptyset, \{i, k\} >$.

Theorem 4.3 Let (X, τ) be an ITS. If $\tilde{A}$ and $\tilde{B}$ are IS of X, then the following properties hold.

- **(i)** $Iint^*(\widetilde{X}_I) = \widetilde{X}_I$
- **(ii)** $Iint^*(\widetilde{\emptyset}_I) = \widetilde{\emptyset}_I$
- **(iii)** $Iint^*(\tilde{A}) \subseteq \tilde{A}$

Proof: Follows from the definition 4.1.

Theorem 4.4 Let (X, τ) be an ITS and $\tilde{A}$ be an IS of X. Then the following properties hold.

- **(i)** $Iint(\tilde{A}) \subseteq Iint^*(\tilde{A}) \subseteq \tilde{A}$.
- **(ii)** If $\tilde{A}$ is Ig – open then $Iint^*(\tilde{A}) = \tilde{A}$.

Proof: (i) $Iint^*(\tilde{A}) \subseteq \tilde{A}$ follows from the theorem 4.3 (iii). Suppose that $\tilde{A}$ is intuitionistic open set. Then $\tilde{A}$ is Ig – open. So {Intuitionistic open set contained in $\tilde{A}$} $\subseteq$ {Ig – open set contained in $\tilde{A}$}. $\cup${ Intuitionistic open set contained in $\tilde{A}$} $\subseteq \cup${ Ig – open set contained in $\tilde{A}$}. That is, $Iint(\tilde{A}) \subseteq Iint^*(\tilde{A})$.
(ii) Follows from definition 4.1 and theorem 4.4 (i).

Remark 4.5 The containment relations in theorem 4.4 (i) may be strict or equal and the converse of the theorem 4.4 (ii) is not true in general as seen from the succeeding examples.

Example 4.6 Let $X = \{i, j, k\}$ and $\tau = \{\widetilde{X}_I, \widetilde{\emptyset}_I, < X, \{k\}, \{i, j\} >, < X, \{i\}, \{k\} >, < X, \{i, k\}, \emptyset > \}$ be an ITS. Let $\tilde{A} = < X, \emptyset, \{k\} >$. Then $Iint(\tilde{A}) = \widetilde{\emptyset}_I$ and $Iint^*(\tilde{A}) = < X, \emptyset, \{j, k\} >$. Therefore $Iint(\tilde{A}) \subset Iint^*(\tilde{A}) \subset \tilde{A}$.

Let $\tilde{A} = < X, \{i\}, \{k\} >$. Then $Iint(\tilde{A}) = < X, \{i\}, \{k\} >$ and $Iint^*(\tilde{A}) = < X, \{i\}, \{k\} >$. Therefore $Iint(\tilde{A}) = Iint^*(\tilde{A}) = \tilde{A}$.

Example 4.7 Let $X = \{i, j, k\}$ and $\tau = \{\widetilde{X}_I, \widetilde{\emptyset}_I, < X, \emptyset, \{i, j\} >, < X, \{i\}, \emptyset >, < X, \{i\}, \{k\} > \}$ be an ITS. Let $\tilde{A} = < X, \emptyset, \{i\} >$. Then $Iint^*(\tilde{A}) = < X, \emptyset, \{i\} > = \tilde{A}$ but $\tilde{A}$ is not Ig – open set.

Theorem 4.8 Let (X, τ) be an ITS and $\tilde{A}, \tilde{B}$ be any two IS of X. Then the following results hold.

- **(i)** If $\tilde{A} \subseteq \tilde{B}$, then $Iint^*(\tilde{A}) \subseteq Iint^*(\tilde{B})$.
- **(ii)** $Iint^*(\tilde{A} \cap \tilde{B}) \subseteq Iint^*(\tilde{A}) \cap Iint^*(\tilde{B})$.

Proof: (i) Let $\tilde{A} \subseteq \tilde{B}$. By definition 4.1 $Iint^*(\tilde{A}) = \cup \{\tilde{E} : \tilde{E}\ is\ Ig - open\ sets\ and\ \tilde{E} \subseteq \tilde{A}\}$. If $\tilde{E} \subseteq \tilde{A}$, then $\tilde{E} \subseteq \tilde{A} \subseteq \tilde{B}$ where $\tilde{E}$ is $Ig – open$. We have $\tilde{E} \subseteq Iint*(\tilde{B})$. Then $Iint^*(\tilde{A}) \subseteq \cup \{\tilde{E} : \tilde{E}\ is\ Ig - open\ sets\ and\ \tilde{E} \subseteq \tilde{B}\} = Iint^*(\tilde{B})$. That is $Iint^*(\tilde{A}) \subseteq Iint^*(\tilde{B})$.
(ii) We have $\tilde{A} \cap \tilde{B} \subseteq \tilde{A}$ and $\tilde{A} \cap \tilde{B} \subseteq \tilde{B}$. Then by (i), $Iint^*(\tilde{A} \cap \tilde{B}) \subseteq Iint^*(\tilde{A})$ and $Iint^*(\tilde{A} \cap \tilde{B}) \subseteq Iint^*(\tilde{B})$. Thus $Iint*(\tilde{A} \cap \tilde{B}) \subseteq Iint^*(\tilde{A}) \cap Iint^*(\tilde{B})$.

Theorem 4.9 Let (X, τ) be an ITS and $\tilde{A}$ be an IS of X. Then

- **(i)** $Icl^*(X - \tilde{A}) = X – Iint^*(\tilde{A})$
- **(ii)** $Iint^*(X - \tilde{A}) = X – Icl^*(\tilde{A})$

Proof: Let $\tilde{p} \in X – Iint^*(\tilde{A})$. Then $\tilde{p} \notin Iint^*(\tilde{A})$. This implies that $\tilde{p}$ does not belong to any $Ig – open$ subset of $\tilde{A}$. Let $\tilde{E}$ be an $Ig – closed$ set containing $X - \tilde{A}$. Then $X - \tilde{E}$ is an $Ig – open$ set contained in $\tilde{A}$. Therefore $\tilde{p} \notin X - \tilde{E}$ and so $\tilde{p} \in \tilde{E}$. Hence $\tilde{p} \in Icl^*(X - \tilde{A})$. Therefore $X – Iint^*(\tilde{A}) \subseteq Icl^*(X - \tilde{A})$.

On the other hand, let $\tilde{p} \in Icl^*(X - \tilde{A})$. Then $\tilde{p}$ belong to every $Ig – closed$ set containing $X - \tilde{A}$. Hence $\tilde{p}$ does not belong to any $Ig – open$ subset of $\tilde{A}$, that is $\tilde{p} \notin Iint^*(\tilde{A})$, then $\tilde{p} \in X – Iint^*(\tilde{A})$. Thus $Icl^*(X - \tilde{A}) \subseteq X – Iint^*(\tilde{A})$. Hence $Icl^*(X - \tilde{A}) = X – Iint^*(\tilde{A})$.
(ii) It can be proved by replacing $\tilde{A}$ by $X - \tilde{A}$ in (i) and using set theoretic properties.

Theorem 4.10 The operator $Iint^*$ is Kuratowski interior operator

Proof: It is similar to the proof of theorem 3.10.

Definition 4.11 If (X, τ) is an ITS, let τ^* be the topology on X defined by the intuitionistic generalized interior operator $Iint^*$. That is, $\tau^* = \{\tilde{A} \in IS(X) : Iint^*(\tilde{G}) = \tilde{G}\}$.

Theorem 4.12 Let (X, τ) be an ITS. Then $\tau \subseteq \tau^*$.

Proof: Let $\tilde{G}$ be any intuitionistic open set. It follows that $\tilde{G}$ is a Ig – open set. Hence $Iint^*(\tilde{G}) = \tilde{G}$, by theorem 4.4 (ii).That is $\tilde{G} \in \tau^*$, and hence $\tau \subseteq \tau^*$.

Theorem 4.13 Let (X, τ) be an ITS. If an IS $\tilde{A}$ of X is Ig – open, then $\tilde{A}$ is τ^* – open.

Proof: It follows from theorem 4.4 (ii) and definition 4.11.

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