Anti Skolem Mean Labeling of Cycle Related Graphs

S. Alice Pappa[1], B. Belsiya[2]

[1]*Department of Mathematics, Nazareth Margoschis College, Pillayanmanai, Nazareth – 628 617, Tamilnadu, INDIA*

[2]*Research Scholar, Reg. No. 19122142092004, Department of Mathematics, Nazareth Margoschis College, Pillayanmanai, Nazareth, Affiliated to Manonmaniam Sundaranar University, Tirunelveli – 627012,Tamilnadu, INDIA*

Abstract:

A graph G = (V, E) with p vertices and q edges where p < q + 1 is said to be an Anti Skolem mean graph if it is possible to label the vertices x ∈ V with distinct labels f(x) from {1, 2, …, q + 1} in such a way that when each edge e = uv is labeled with $f(e = uv) = \frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd then the resulting edges are distinct labels from the set *{2, 3, . . ., p}*. In this case *f* is called an Anti Skolem Mean labeling of G. In this paper, we prove that Cycle Related Graphs are Anti Skolem Mean graphs.

Keywords: Anti Skolem Mean Graph, Anti Skolem Mean labeling, Cycle related graphs.

1. Introduction

All graphs with p vertices and q edges are finite, simple and undirected graph without loops or parallel edges. Detailed survey for all graph labeling we refer to Gallian [1]. For all other standard terminology and notations, we follow Harary [2]. The concept of Skolem Mean Labeling was introduced by A. Subramanian, D. S.T. Ramesh and V. Balaji [4]. In this paper we introduced the new concept Anti Skolem Mean labeling and we investigate it for cycle related graphs.

Definition: 1.1

A graph G = (V, E) with p vertices and q edges where p < q + 1 is said to be an Anti Skolem Mean graph if it is possible to label the vertices x ∈V with distinct labels f(x) from {1, 2, … ,q + 1} in such a way that when each edge e = uv is labeled with $f(e = uv) = \frac{f(u)+f(v)}{2}$ if f(u) + f(v) is even and $\frac{f(u)+f(v)+1}{2}$ if f(u) + f(v) is odd then the resulting edges are distinct labels from the set *{2, 3, . . ., p}*.

Result 1.2

The only graph satisfies the condition $p < q + 1$ are cycle graphs.

Definition 1.3

A walk in which $u_1, u_2, \ldots, u_n$ are distinct is called a path. A path on n vertices is denoted by P_n.

Definition 1.4

A closed path is called a cycle. A cycle on n vertices is denoted by C_n.

Definition1.5

The Dumbbell graph $D_{n,m}$ is obtained by joining two disjoint cycle with an edge.

2. **Main Results**

Theorem 2. 1

Any cycle C_n is an Anti Skolem Mean graph for all $n \geq 3$.

Proof:

Let C_n be the cycle with vertices $u_1, u_2, \ldots, u_n$.

Here we consider two different cases:

Case (i): If n is odd

Define a function $f: V(C_n) \rightarrow \{1,2,\ldots, q + 1\}$ by

$$f(u_i) = \begin{cases} 2i - 1, & 1 \leq i \leq \frac{n+1}{2}, \\ 2(n - i + 2), & \frac{n+3}{2} \leq i \leq n, \end{cases}$$

Then the edges are labeled as

$$f(u_i u_{i+1}) = \begin{cases} 2i, & 1 \leq i \leq \frac{n+1}{2}, \\ 2(n - i) + 3, & \frac{n+3}{2} \leq i \leq n. \end{cases}$$

Then the edge labels are distinct.

Hence, Cycle C_n is an Anti Skolem Mean Graph.

Example 2.2 Anti Skolem Mean labeling of cycle C_7 is given below

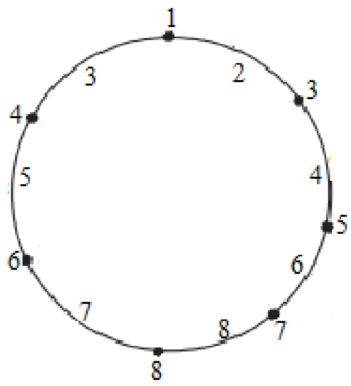

Figure: 1 Cycle C_7

Case (ii): If n is even

Define a function f: $V(C_n) \rightarrow \{1,2,\ldots, q+1\}$ by

$$f(u_i) = \begin{cases} 2i - 1, & 1 \le i \le \frac{n+2}{2}, \\ 2(n - i + 2), & \frac{n+4}{2} \le i \le n, \end{cases}$$

Then the edges are labeled as

$$f(u_i u_{i+1}) = \begin{cases} 2i, & 1 \le i \le \frac{n}{2}, \\ 2(n - i) + 3, & \frac{n+2}{2} \le i \le n. \end{cases}$$

Then the edge labels are distinct.

Hence, Cycle C_n is an Anti Skolem Mean Graph.

Example 2. 3 Anti Skolem Mean labeling of cycle C_8 is given below

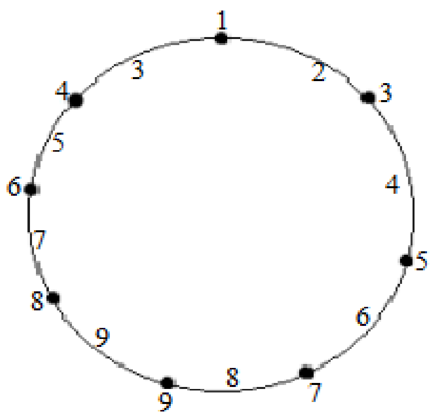

Figure: 2 Cycle C_8

Hence, from case (i) and case (ii), we conclude that cycle C_n is an Anti Skolem Mean graph for all $n \ge 3$.

Theorem 2.4

The graph $C_n^{(2)}$ is an Anti Skolem Mean graph for all $n \geq 3$.

Proof:

Let u_i, $1 \leq i \leq n$ be the vertices of first cycle of $C_n^{(2)}$.

Let v_i, $1 \leq i \leq n$ be the vertices of second cycle of $C_n^{(2)}$.

Here we consider two different cases:

Case (i): If n is odd.

Here $u_{\frac{n+3}{2}} = v_1$ be the central vertex of $C_n^{(2)}$.

Define a function $f: V(C_n^{(2)}) \rightarrow \{1,2,\ldots, q+1\}$ by

$$f(u_i) = \begin{cases} 2i - 1, & 1 \leq i \leq \frac{n+1}{2}, \\ 2(n - i + 2), & \frac{n+3}{2} \leq i \leq n, \end{cases}$$

$$f(v_i) = \begin{cases} n + 2i - 1, & 1 \leq i \leq \frac{n+1}{2}, \\ n + 2(n - i + 2), & \frac{n+3}{2} \leq i \leq n, \end{cases}$$

Then the edges are labeled as

$$f(u_iu_{i+1}) = \begin{cases} 2i, & 1 \leq i \leq \frac{n+1}{2}, \\ 2(n - i) + 3, & \frac{n+3}{2} \leq i \leq n, \end{cases}$$

$$f(v_iv_{i+1}) = \begin{cases} n + 2i, & 1 \leq i \leq \frac{n+1}{2}, \\ n + 2(n - i) + 3, & \frac{n+3}{2} \leq i \leq n. \end{cases}$$

Then the edge labels are distinct.

Hence, $C_n^{(2)}$ is an Anti Skolem Mean Graph.

Example 2.5 Anti Skolem Mean labeling of $C_5^{(2)}$ is given below.

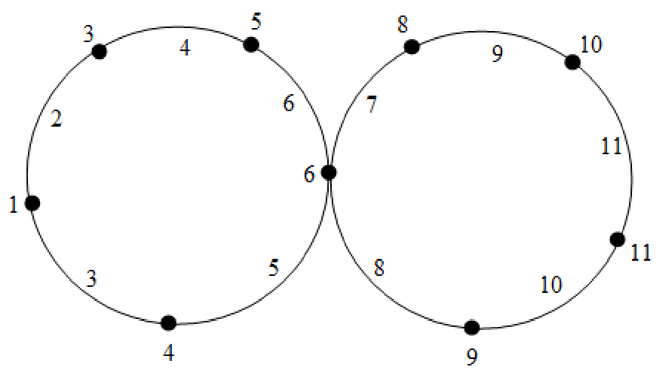

Figure: 3 $C_5^{(2)}$

Case (ii): If n is even.

Here $u_{\frac{n+2}{2}} = v_1$ be the central vertex of $C_n^{(2)}$.

Define a function f: $V(C_n^{(2)}) \to \{1,2,\dots, q + 1\}$ by

$$f(u_i) = \begin{cases} 2i - 1, & 1 \le i \le \frac{n+2}{2}, \\ 2(n - i + 2), & \frac{n+4}{2} \le i \le n, \end{cases}$$

$$f(v_i) = \begin{cases} n + 2i - 1, & 1 \le i \le \frac{n+2}{2}, \\ n + 2(n - i + 2), & \frac{n+4}{2} \le i \le n, \end{cases}$$

Then the edges are labeled as

$$f(u_iu_{i+1}) = \begin{cases} 2i, & 1 \le i \le \frac{n}{2}, \\ 2(n - i) + 3, & \frac{n+2}{2} \le i \le n, \end{cases}$$

$$f(v_iv_{i+1}) = \begin{cases} n + 2i, & 1 \le i \le \frac{n}{2}, \\ n + 2(n - i) + 3, & \frac{n+2}{2} \le i \le n. \end{cases}$$

Then the edge labels are distinct.

Hence, $C_n^{(2)}$ is an Anti Skolem Mean Graph.

Example 2.6 Anti Skolem Mean labeling of $C_6^{(2)}$ is given below.

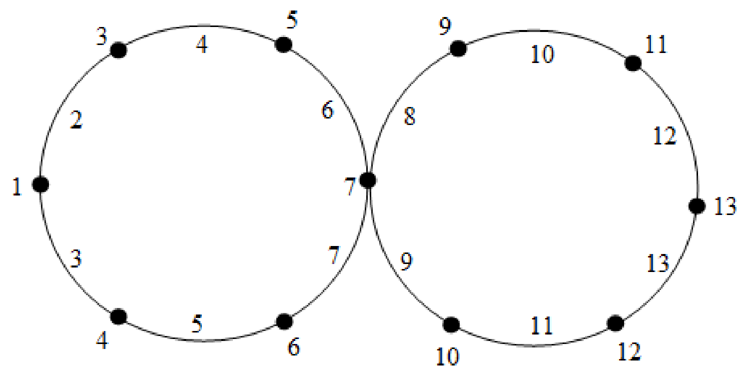

Figure: 4 $C_6^{(2)}$

Hence, from case (i) and case (ii), we conclude that $C_n^{(2)}$ is an Anti Skolem Mean graph for all $n \ge 3$.

Theorem 2.7

The Dumbbell graph $D_{n,m}$ is an Anti Skolem Mean graph for all $n, m \ge 3$.

Proof:

Let u_i, $1 \le i \le n$ be the vertices of first cycle C_n of Dumbbell graph $D_{n,m}$.

Let v_i, $1 \le i \le m$ be the vertices of second cycle C_m of Dumbbell graph $D_{n,m}$.

Here we consider four different cases:

Case (i): If n is odd and m is even.

Let $u_{\frac{n+3}{2}}v_1$ be the edge joining two disjoint cycle of Dumbbell graph D $_{n,m}$.

Define a function f: V(D $_{n, m}$) → {1,2,…, q + 1} by

$$f(u_i) = \begin{cases} 2i - 1, & 1 \le i \le \frac{n+1}{2}, \\ 2(n - i + 2), & \frac{n+3}{2} \le i \le n, \end{cases}$$

$$f(v_i) = \begin{cases} n + 2i, & 1 \le i \le \frac{m+2}{2}, \\ n + 2(n - i + 2) + 1, & \frac{m+4}{2} \le i \le m, \end{cases}$$

Then the edges are labeled as

$$f(u_iu_{i+1}) = \begin{cases} 2i, & 1 \le i \le \frac{n+1}{2}, \\ 2(n - i) + 3, & \frac{n+3}{2} \le i \le n, \end{cases}$$

$f(u_{\frac{n+3}{2}}v_1)$ = n + 2,

$$f(v_iv_{i+1}) = \begin{cases} n + 2i + 1, & 1 \le i \le \frac{m}{2}, \\ n + 2(n - i) + 3, & \frac{m+2}{2} \le i \le m. \end{cases}$$

Then the edge labels are distinct.

Hence, Dumbbell graph D $_{n, m.}$ is an Anti Skolem Mean Graph.

Example 2.8 Anti Skolem Mean labeling of Dumbbell graph D $_{5, 6}$ is given below.

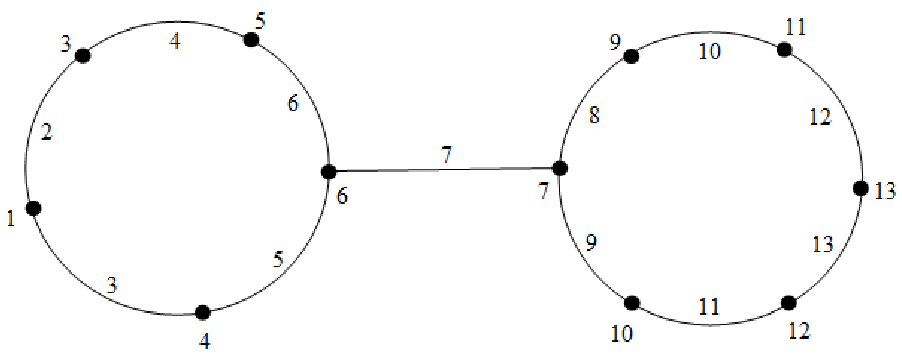

Figure: 5 Dumbbell graph D $_{5, 6}$

Case (ii): If n is even and m is odd.

Let $u_{\frac{n+2}{2}}v_1$ be the edge joining two disjoint cycle of Dumbbell graph D $_{n, m}$.

Define a function f: V(D $_{n, m}$) → {1,2,…, q + 1} by

$$f(u_i) = \begin{cases} 2i - 1, & 1 \le i \le \frac{n+2}{2}, \\ 2(n - i + 2), & \frac{n+4}{2} \le i \le n, \end{cases}$$

$$f(v_i) = \begin{cases} n + 2i, & 1 \le i \le \frac{m+1}{2}, \\ n + 2(n - i + 2) + 1, & \frac{m+3}{2} \le i \le m, \end{cases}$$

Then the edges are labeled as

$$f(u_i u_{i+1}) = \begin{cases} 2i, & 1 \le i \le \frac{n}{2}, \\ 2(n - i) + 3, & \frac{n+2}{2} \le i \le n, \end{cases}$$

$f(u_{\frac{n+2}{2}} v_1) = n + 2,$

$$f(v_i v_{i+1}) = \begin{cases} n + 2i + 1, & 1 \le i \le \frac{m+1}{2}, \\ n + 2(n - i) + 3, & \frac{m+3}{2} \le i \le m. \end{cases}$$

Then the edge labels are distinct.

Hence, Dumbbell graph $D_{n, m}$ is an Anti Skolem Mean Graph.

Example 2.9 Anti Skolem Mean labeling of Dumbbell graph $D_{6, 5}$ is given below.

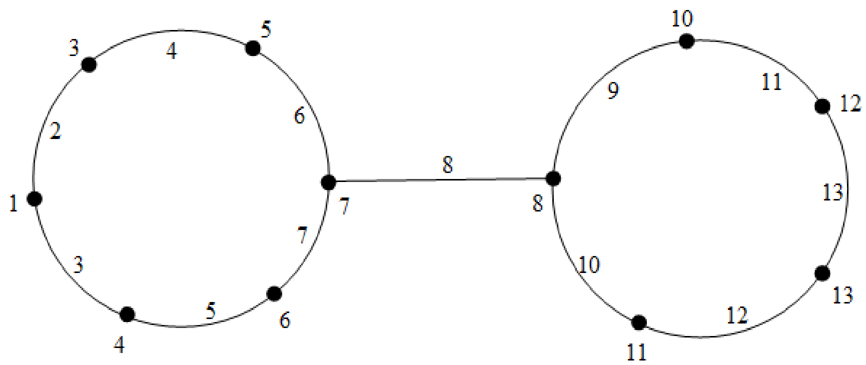

Figure: 6 Dumbbell graph $D_{6, 5}$

Case (iii): If n is odd and m is odd.

Let $u_{\frac{n+3}{2}} v_1$ be the edge joining two disjoint cycle of Dumbbell graph $D_{n,m}$.

Define a function f: V($D_{n,m}$) → {1,2,…, q + 1} by

$$f(u_i) = \begin{cases} 2i - 1, & 1 \le i \le \frac{n+1}{2}, \\ 2(n - i + 2), & \frac{n+3}{2} \le i \le n, \end{cases}$$

$$f(v_i) = \begin{cases} n + 2i, & 1 \le i \le \frac{m+1}{2}, \\ n + 2(n - i + 2) + 1, & \frac{m+3}{2} \le i \le m, \end{cases}$$

Then the edges are labeled as

$$f(u_iu_{i+1}) = \begin{cases} 2i, & 1 \leq i \leq \frac{n+1}{2}, \\ 2(n-i)+3, & \frac{n+3}{2} \leq i \leq n, \end{cases}$$

$f(u_{\frac{n+3}{2}}v_1) = n + 2,$

$$f(v_iv_{i+1}) = \begin{cases} n+2i+1, & 1 \leq i \leq \frac{m+1}{2}, \\ n+2(n-i)+3, & \frac{m+3}{2} \leq i \leq m. \end{cases}$$

Then the edge labels are distinct.

Hence, Dumbbell graph $D_{n,m}$ is an Anti Skolem Mean Graph.

Example 2.10 Anti Skolem Mean labeling of Dumbbell graph $D_{5,5}$ is given below.

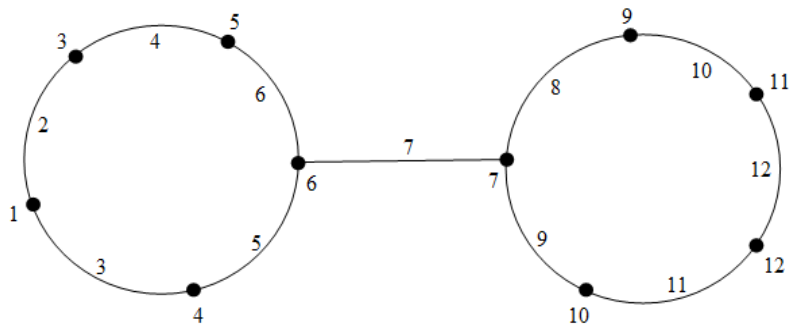

Figure: 7 Dumbbell graph $D_{5,5}$

Case (iv): If n is even and m is even.

Let $u_{\frac{n+2}{2}}v_1$ be the edge joining two disjoint cycle of Dumbbell graph $D_{n,m}$.

Define a function f: $V(D_{n,m}) \rightarrow \{1,2,\ldots, q+1\}$ by

$$f(u_i) = \begin{cases} 2i-1, & 1 \leq i \leq \frac{n+2}{2}, \\ 2(n-i+2), & \frac{n+4}{2} \leq i \leq n, \end{cases}$$

$$f(v_i) = \begin{cases} n+2i, & 1 \leq i \leq \frac{m+2}{2}, \\ n+2(n-i+2)+1, & \frac{m+4}{2} \leq i \leq m, \end{cases}$$

Then the edges are labeled as

$$f(u_iu_{i+1}) = \begin{cases} 2i, & 1 \leq i \leq \frac{n}{2}, \\ 2(n-i)+3, & \frac{n+2}{2} \leq i \leq n, \end{cases}$$

$f(u_{\frac{n+2}{2}}v_1) = n + 2,$

$$f(v_iv_{i+1}) = \begin{cases} n+2i+1, & 1 \leq i \leq \frac{m}{2}, \\ n+2(n-i)+3, & \frac{m+2}{2} \leq i \leq m. \end{cases}$$

Then the edge labels are distinct.

Hence, Dumbbell graph $D_{n, m}$ is an Anti Skolem Mean Graph.

Example 2.11 Anti Skolem Mean labeling of Dumbbell graph $D_{6, 6}$ is given below.

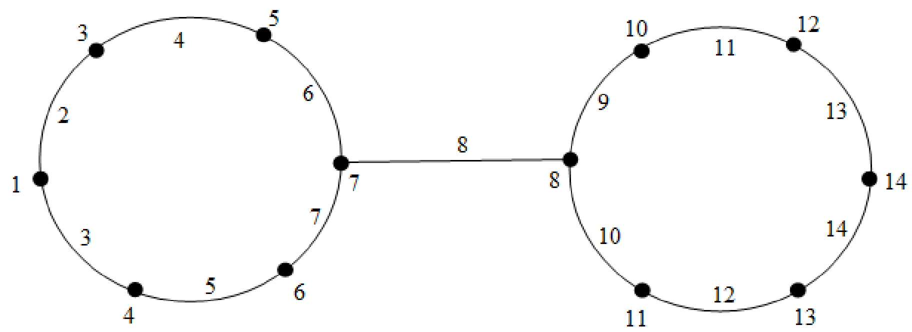

Figure: 8 Dumbbell graph $D_{6, 6}$

Hence, from case (i), case (ii), case (iii) and case (iv), we conclude that Dumbbell graph $D_{n, m}$ is an Anti Skolem Mean graph for n, m ≥ 3.

Conclusion

All graphs are not Anti Skolem Mean graphs. It is very interesting to investigate graphs which admit Anti Skolem Mean labeling. In this paper, we proved that Cycle related graphs are Anti Skolem Mean graphs. It is possible to investigate similar results for several other graphs.

Acknowledgements. The authors thank the referees for their comments and valuable suggestions.

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