INTERIOR DOMINATING SETS AND INTERIOR DOMINATION POLYNOMIALS OF CENTIPEDES

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Abstract: Let $G = (V, E)$ be a undirected graph, without loop and multiple edges. Let P_n^* be the centipede with $2n$ vertices. P_n^* is $P_n \circ K_1$. We denote the graph granted from P_n^* by deleting the vertex labeled $2n$ as $P_n^* - \{2n\}$. Let $D_{Id}(P_n^*, j)$ and $D_{Id}(P_n^* - \{2n\}, j)$ be the family of interior dominating sets of G with cardinality j. Let $d_{Id}(P_n^*, j) = |D_{Id}(P_n^*, j)|$ and $d_{Id}(P_n^* - \{2n\}, j) = |D_{Id}(P_n^* - \{2n\}, j)|$. In this paper, we grant a recursive formula for $d_{Id}(P_n^*, j)$ and $d_{Id}(P_n^*, j)$. Using this recursive formula, we create the polynomial $D_{Id}(P_n^*, x) = d_{Id}(P_n^*, j)x^j$ where $j = n$ and also we create a polynomial $D_{Id}(P_n^* - \{2n\},, x) = d_{Id}(P_n^* - \{2n\}, j)x^j$ where $j = n - 1$ which we investigate interior domination polynomial of P_n^* , $P_n^* - \{2n\}$ and grant some properties of this polynomial.

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Keywords: Centipedes, interior dominating sets, interior domination polynomial.

1. INTRODUCTION

Let $G = (V, E\ )$ be a undirected graph, without loop and multiple edges. A non empty set $D \subseteq V$ is a dominating set of G, if every vertex $V - D$ is adjacent to minimum one vertex in D. The cardinality of minimum dominating set is named as the domination number and is denoted by $\gamma(G)$[4]. A vertex v is an interior vertex of G if for every vertex u distinct from v, there exists a vertex w such that v lies between u and w. A set $D \subseteq V(G)$ is an interior dominating set if D is a dominating set of G and every vertex v interior vertex of G. Any pendent vertices will not be a member in interior set of a graph. $\gamma_{Id}(G)$ denoted as the cardinality of minimum interior dominating set. Let P_n^* be the centipede with $2n$ vertices get by adjoining a single pendent edge to each vertex of a path P_n. Write that P_n^* is $P_n \circ K_1$. We denote the graph granted from P_n^* by deleting the vertex labeled $2n$ as $P_n^* - \{2n\}$ [3]. In this paper we investigate interior domination polynomial of P_n^* , $P_n^* - \{2n\}$ and grant some properties of this polynomial.

Example 1.1

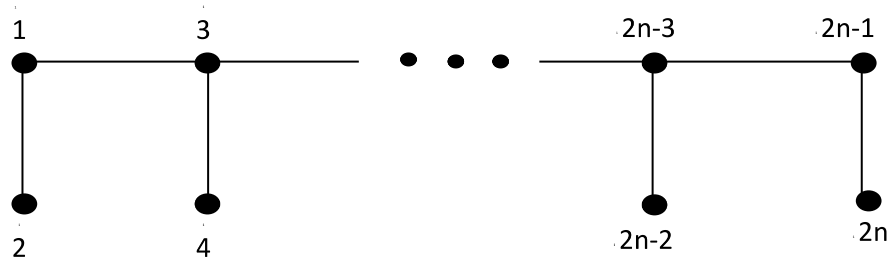

Figure 1. Centipede $\boldsymbol{P_n^*}$

2. Interior Dominating Sets Of $\boldsymbol{P_n^* - \{2n\}}$

Lemma: 2. 1 Let $P_n^* - \{2n\}$ be the centipede with $2n-1$ vertices for every $n \geq 2$.
(a) $\gamma_{Id}(P_n^* - \{2n\}) = \mathrm{n} - 1$

(b) $D_{Id}(P_n^* - \{2n\}, j)$ is empty iff $j > n-1$ or $j < \mathrm{n} - 1$
(c) $D_{Id}(P_n^* - \{2n\}, j)$ is non empty iff $j = n-1$

Lemma (2.2) and (2.3) follows from the lemma 2.1 (b) and (c)
Lemma 2.2 If $D_{Id}(P_n^* - \{2n\}, j-1) = \varphi$ and $D_{Id}(P_{n-1}^* - \{2n-2\}, j-1) \neq \varphi$ then $D_{Id}(P_n^* - \{2n\}, j) \neq \varphi$.
Lemma 2.3 If $D_{Id}(P_n^* - \{2n\}, j) \neq \varphi$ then $D_{Id}(P_n^* - \{2n\}, j-1) = \varphi$ and $D_{Id}(P_{n-1}^* - \{2n-2\}, j-1) \neq \varphi$ iff $j = n-1$.
Theorem 2.4 For every $n \geq 2$, $D_{Id}(P_n^* - \{2n\}, j-1) = \varphi$ and $D_{Id}(P_{n-1}^* - \{2n-2\}, j-1) \neq \varphi$ then $D_{Id}(P_n^* - \{2n\}, j) = \{\{1,3,5, \dots, 2n-3\}\}$.
Proof: Since $D_{Id}(P_n^* - \{2n\}, j-1) = \varphi$ and $D_{Id}(P_{n-1}^* - \{2n-2\}, j-1) \neq \varphi$
From lemma (2.3) we have $j = n-1$
Therefore $D_{Id}(P_n^* - \{2n\}, j) = D_{Id}(P_n^* - \{2n\}, n-1) = \{\{1,3,5, \dots, 2n-3\}\}$.

3. Interior Dominating Sets Of $\boldsymbol{P_n^*}$

Lemma: 3. 1 Let P_n^* be the centipede with $2n$ vertices for every $n \geq 2$.
(a) $\gamma_{Id}(P_n^*\}) = \mathrm{n}$
(b) $D_{Id}(P_n^*, j)$ is empty iff $j > n$ or $j < \mathrm{n}$
(c) $D_{Id}(P_n^*, j)$ is non empty iff $j = n$

Lemma (3.2) and (3.3) follows from the lemma 3.1 (b) and (c)
Lemma 3.2 If $D_{Id}(P_n^*, j-1) = \varphi$ and $D_{Id}(P_{n-1}^*, j-1) \neq \varphi$ then $D_{Id}(P_n^*, j) \neq \varphi$.
Lemma 3.3 If $D_{Id}(P_n^*, j) \neq \varphi$ then $D_{Id}(P_n^*, j-1) = \varphi$ and $D_{Id}(P_{n-1}^*, j-1) \neq \varphi$ iff $j = n$.
Theorem 3.4 For every $n \geq 2$, $D_{Id}(P_n^*, j-1) = \varphi$ and $D_{Id}(P_{n-1}^*, j-1) \neq \varphi$ then $D_{Id}(P_n^*, j) = \{\{1,3,5, \dots, 2n-1\}\}$.
Proof: Since $D_{Id}(P_n^*, j-1) = \varphi$ and $D_{Id}(P_{n-1}^*, j-1) \neq \varphi$

From lemma (3.3) we have $j = n$
Therefore $D_{Id}(P_n^*, j)$ = $D_{Id}(P_n^*, n)$ = $\{\{1,3,5, \dots, 2n-1\}\}$.

4. Interior Domination Polynomials Of $\mathbf{P_n^* - \{2n\}}$

Definition 4.1 Let $D_{Id}(P_n^* - \{2n\}, j)$ be the family of interior dominating sets of a centipede $P_n^* - \{2n\}$ with cardinality j and Let $d_{Id}(P_n^* - \{2n\}, j) = |D_{Id}(P_n^* - \{2n\}, j)|$. Then the interior domination polynomial $D_{Id}(P_n^* - \{2n\}, x)$ of $P_n^* - \{2n\}$ is defined as $D_{Id}(P_n^* - \{2n\}, x) = d_{Id}(P_n^* - \{2n\}, j)x^j$ where $j = n - 1$

Theorem 4.2

(a) If $D_{Id}(P_n^* - \{2n\}, x)$ is the family of interior dominating sets with cardinality j of $P_n^* - \{2n\}$ then
$d_{Id}(P_n^* - \{2n\}, j)$ = $d_{Id}(P_{n-1}^* - \{2n-2\}, j-1)$
where $d_{Id}(P_n^* - \{2n\}, j) = |D_{Id}(P_n^* - \{2n\}, j)|$.
(b) For every ≥ 2 , $D_{Id}(P_n^* - \{2n\}, x) = x[D_{Id}(P_{n-1}^* - \{2n-2\}, x)]$

Proof

(a) From theorem (2.4) we have $D_{Id}(P_n^* - \{2n\}, j) = \{\{1,3,5, \dots, 2n-3\}\}$.
Therefore $|D_{Id}(P_n^* - \{2n\}, j)|$ = $|D_{Id}(P_{n-1}^* - \{2n-2\}, j-1)|$
Therefore $d_{Id}(P_n^* - \{2n\}, j)$ = $d_{Id}(P_n^* - \{2n-2\}, j-1)$
where $d_{Id}(P_n^* - \{2n\}, j) = |D_{Id}(P_n^* - \{2n\}, j)|$.

(b) We have $d_{Id}(P_n^* - \{2n\}, j)$ = $d_{Id}(P_n^* - \{2n-2\}, j-1)$
$d_{Id}(P_n^* - \{2n\}, j)x^j$ = $d_{Id}(P_n^* - \{2n-2\}, j-1)x^j$
$\Sigma d_{Id}(P_n^* - \{2n\}, j)x^j$ = $\Sigma d_{Id}(P_n^* - \{2n-2\}, j-1)x^j$
= $x[D_{Id}(P_{n-1}^* - \{2n-2\}, x)]$.

5. Interior Domination Polynomials Of $\mathbf{P_n^*}$

Definition 5.1 Let $D_{Id}(P_n^*, j)$ be the family of interior dominating sets of a centipede P_n^* with cardinality j and Let $d_{Id}(P_n^*, j) = |D_{Id}(P_n^*, j)|$. Then the interior domination polynomial $D_{Id}(P_n^*, x)$of P_n^* is defined as $D_{Id}(P_n^*, x) = d_{Id}(P_n^*, j)x^j$ where $j = n$

Theorem 5.2

(a) If $D_{Id}(P_n^*, x)$ is the family of interior dominating sets with cardinality j of P_n^*
then $d_{Id}(P_n^*, j)$ = $d_{Id}(P_{n-1}^*, j-1)$
where $d_{Id}(P_n^*, j) = |D_{Id}(P_n^*, j)|$.
(b) For every ≥ 2 , $D_{Id}(P_n^*, x) = x[D_{Id}(P_{n-1}^*, x)]$

Proof

(a) From theorem (3.4) we have $D_{Id}(P_n^*, j) = \{\{1,3,5, \dots, 2n-1\}\}$.
Therefore $|D_{Id}(P_n^*, j)|$ = $|D_{Id}(P_{n-1}^*, j-1)|$
Therefore $d_{Id}(P_n^*, j)$ = $d_{Id}(P_{n-1}^*, j-1)$
where $d_{Id}(P_n^*, j) = |D_{Id}(P_n^*, j)|$.

(b) We have $d_{Id}(P_n^*, j)$ = $d_{Id}(P_{n-1}^*, j-1)$
$d_{Id}(P_n^*, j)x^j$ = $d_{Id}(P_{n-1}^*, j-1)x^j$
$\Sigma d_{Id}(P_n^*, j)x^j$ = $\Sigma d_{Id}(P_{n-1}^*, j-1)x^j$
= $x[D_{Id}(P_{n-1}^*, x)]$.

We obtain $d_{Id}\left(P_n^*, j\right)$ and $d_{Id}\left(P_n^* - \{2n\}, j\right)$ for $2 \leq n \leq 5$ as shown in table (1)

Table 1 : (P_n^*, j) and $d_{Id}(P_n^* - \{2n\}, j)$

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j/n	1	2	3	4	5
$P_2^* - \{4\}$	1				
P_2^*	0	1			
$P_3^* - \{6\}$	0	1			
P_3^*	0	0	1		
$P_4^* - \{8\}$	0	0	1		
P_4^*	0	0	0	1	
$P_5^* - \{10\}$	0	0	0	1	
P_5^*	0	0	0	0	1

In the following theorem we include some properties of $d_{Id}\left(P_n^*, j\right)$ and $d_{Id}\left(P_n^* - \{2n\}, j\right)$.

Theorem 5.3
The following properties hold for the coefficients of $D_{Id}\left(P_n^*, j\right)$ and $D_{Id}\left(P_n^* - \{2n\}, j\right)$.
(i) $d_{Id}\left(P_n^*, n\right) = 1$, for every $n \geq 2$
(ii) $d_{Id}\left(P_n^* - \{2n\}, n-1\right) = 1$, for every $n \geq 2$

Proof
Proof is obvious

6. Conclusion
In this paper we have described the interior dominating sets and some properties of interior domination polynomials of centipedes.

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