Secure Dominating Sets and Secure Domination Polynomials of Centipedes

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Abstract- Let $G = (V, E)$ be a simple graph. A dominating set S of G is a secure dominating set if for each $u \in V - S$ there exists $v \in N(u) \cap S$ such that $(S - \{v\} \cup \{u\})$ is a dominating set. Let P_n^* be the centipede with $2n$ vertices and let $\mathcal{D}_s(P_n^*, i)$ denote the family of all secure dominating sets of P_n^* with cardinality i. Let $d_s(P_n^*, i) = |\mathcal{D}_s(P_n^*, i)|$. In this paper, we obtain recursive formula for $d_s(P_n^*, i)$. Using this recursive formula, we construct the polynomial, $D_s(P_n^*, x) = \sum_{i=n}^{2n} d_s(P_n^*, i)x^i$ which we call secure domination polynomial of P_n^* and obtain some properties of this polynomial.

Index Terms- domination, secure domination, secure domination number, secure dominating set, secure domination polynomial.

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I. Introduction

By a graph $G = (V, E)$, we mean a finite, undirected graph with neither loops nor multiple edges. The order $|V|$ and the size $|E|$ of G are denoted by n and m respectively. For graph theoretic terminology we refer to Chartrand and Lesniak [3]. For any vertex $v \in V$, the open neighborhood of v is the set $N(v) = \{u \in V / uv \in E\}$ and the closed neighborhood of v is the set $N[v] = N(v) \cup \{v\}$. For a set $S \subseteq V$, the open neighborhood of S is $N(S) = \bigcup_{v\in S} N(v)$ and the closed neighborhood of S is $N[S] = N(S) \cup S$. A set $S \subseteq V$ is a dominating set of G, if $N[S] = V$, or equivalently, every vertex in $V - S$ is adjacent to at least one vertex in S. A dominating set S of G is a secure dominating set if for each $u \in V - S$ there exists $v \in N(u) \cap S$ such that $(S - \{v\}) \cup \{u\}$ is a dominating set. In this case we say that u is S- defended by v or v S-defends u. The secure domination number $\gamma_s(G)$ is the minimum cardinality of a secure dominating set. The concept secure dominating set is introduced by Cockayne et al [4]. A simple path is a path in which all its internal vertices have degree two, and the end vertices have degree one and is denoted by P_n. Let P_n^* denotes the centipede with $2n$ vertices obtained by appending a single pendant edge to each vertex of a path P_n. For the definition of centipede, we refer S. Alikhani and Y-H. Peng [2].

Definition 1.1[9].

Let G be a simple connected graph. Let $\mathcal{D}_s(G, i)$ denote the family of all secure dominating set of G with cardinality i and let $d_s(G, i) = |\mathcal{D}_s(G, i)|$. Then the secure domination polynomial $D_s(G, x)$ of G is defined as $\mathcal{D}_s(G, x) = \sum_{i=\gamma_s(G)}^{|V(G)|} d_s(G, i)x^i$, where $\gamma_s(G)$ is the secure domination number of G.

As usual we use $\lfloor x \rfloor$ for the largest integer less than or equal to x and $\lceil x \rceil$ for the smallest integer greater than or equal to x. Also, we denote the set $\{1,2, \dots, n\}$ by $[n]$, throughout this paper.

In the next section we study secure dominating sets and secure domination polynomial of $P_n^* - \{2n\}$, which is needed for the study of secure dominating sets of centipedes.

II. SECURE DOMINATING SETS AND SECURE DOMINATION POLYNOMIOAL OF $P_n^* - \{2n\}$

Lemma 2.1
For every $n \in \mathbb{N}$
i) $\gamma_s(P_n^*) = n$
ii) $\gamma_s(P_n^* - \{2n\}) = n$
iii) $\mathcal{D}_s(P_n^*, i) = \emptyset$ if and only if $i < n$ or $i > 2n$
iv) $\mathcal{D}_s(P_n^* - \{2n\}, i) = \emptyset$ if and only if $i < n$ or $i > 2n - 1$.

Lemma 2.2
i) If $\mathcal{D}_s(P_{n-1}^*, i - 1) \neq \emptyset$ and $\mathcal{D}_s(P_{n-2}^*, i - 2) = \emptyset$, then $\mathcal{D}_s(P_{n-1}^* - \{2n - 2\}, i - 2) \neq \emptyset$.
ii) If $\mathcal{D}_s(P_{n-1}^*, i - 1) \neq \emptyset$ and $\mathcal{D}_s(P_{n-1}^* - \{2n - 2\}, i - 2) = \emptyset$, then $\mathcal{D}_s(P_{n-2}^*, i - 2) \neq \emptyset$.
iii) If $\mathcal{D}_s(P_{n-1}^* - \{2n - 2\}, i - 2) \neq \emptyset$ and $\mathcal{D}_s(P_{n-2}^*, i - 2) \neq \emptyset$, then $\mathcal{D}_s(P_{n-1}^*, i - 1) \neq \emptyset$.

Proof:
i) Since $\mathcal{D}_s(P_{n-1}^*, i - 1) \neq \emptyset$, by Lemma 2.1(iii), $n - 1 \le i - 1 \le 2n - 2$.
$\Rightarrow n - 2 \le i - 2 \le 2n - 3$
Also $n - 2 < n - 1$. Therefore, by Lemma 2.1(iv), $\mathcal{D}_s(P_{n-1}^* - \{2n - 2\}, i - 2) \neq \emptyset$.
ii) Since $\mathcal{D}_s(P_{n-1}^*, i - 1) \neq \emptyset$, by Lemma 2.1(iii), $n - 1 \le i - 1 \le 2n - 2$.
$\Rightarrow n - 2 \le i - 2$ (1)
Since $\mathcal{D}_s(P_{n-1}^* - \{2n - 2\}, i - 2) = \emptyset$, by Lemma 2.1(iv), $i - 2 < n - 1$ or $i - 2 > 2n - 3$.
$\Rightarrow i - 2 \le n - 1$ (2)
From (1) and (2), we have $i - 2 = n - 2$. By Lemma 2.1(iii), $\mathcal{D}_s(P_{n-2}^*, i - 2) \neq \emptyset$.

iii) Since $\mathcal{D}_s(P^*_{n-2}, i-2) \neq \emptyset$, by Lemma 2.1(iii), $n-2 \leq i-2 \leq 2n-4$.
$\Rightarrow n-1 \leq i-1 \leq 2n-3$ (3)
Since $\mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) \neq \emptyset$, by Lemma 2.1(iv), $n-1 \leq i-2 \leq 2n-3$.
$\Rightarrow n \leq i-1 \leq 2n-2$ (4)
From (3) and (4), we have
$n-1 \leq i-1 \leq 2(n-1)$.
By Lemma2.1(iii), $\mathcal{D}_s(P^*_{n-1}, i-1) \neq \emptyset$.

Lemma 2.3

For every $n \geq 3$,

i) $\mathcal{D}_s(P^*_{n-1}, i-1) \neq \emptyset$, $\mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) \neq \emptyset$ and $\mathcal{D}_s(P^*_{n-2}, i-2) = \emptyset$ if and only if $i = 2n-1$.

ii) $\mathcal{D}_s(P^*_{n-1}, i-1) \neq \emptyset$, $\mathcal{D}_s(P^*_{n-2}, i-2) \neq \emptyset$ and $\mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) = \emptyset$ if and only if $i = n$.

iii) $\mathcal{D}_s(P^*_{n-1}, i-1) \neq \emptyset$, $\mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) \neq \emptyset$ and $\mathcal{D}_s(P^*_{n-2}, i-2) \neq \emptyset$ if and only if
$n+1 \leq i \leq 2n-2$.

Proof:

i) ($\Rightarrow$) Since $\mathcal{D}_s(P^*_{n-1}, i-1) \neq \emptyset$, by Lemma 2.1(iii), $n-1 \leq i-1 \leq 2n-2$.
$\Rightarrow i-1 \leq 2n-2$
$\Rightarrow i \leq 2n-1$ (5)
Since $\mathcal{D}_s(P^*_{n-2}, i-2) = \emptyset$, by Lemma 2.1(iii),
$i-2 < n-2$ or $i-2 > 2n-4$.
$\Rightarrow i < n$ or $i > 2n-2$. (6)
From (5) and (6), $2n-2 < i \leq 2n-2$.
Hence $i = 2n-1$.
($\Leftarrow$) Suppose $i = 2n-1$.
Then $i-2 = 2n-3 > 2(n-2)$. By Lemma 2.1(iii), $\mathcal{D}_s(P^*_{n-2}, i-2) = \emptyset$.
Since $i-1 = 2(n-1)$, by Lemma 2.1(iii), $\mathcal{D}_s(P^*_{n-1}, i-1) \neq \emptyset$.
Since $i-2 = 29n-1)-1$, by Lemma 2.1(iv), $\mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) \neq \emptyset$.

ii) ($\Rightarrow$) Since $\mathcal{D}_s(P^*_{n-1}, i-1) \neq \emptyset$, by Lemma 2.1(iii), $n-1 \leq i-1 \leq 2n-2$.
$\Rightarrow n \leq i$ (7)
Since $\mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) = \emptyset$, by Lemma 2.1(iv), $i-2 < n-1$ or $i-2 > 2n-3$.
$\Rightarrow i < n+1$ (8)
From (7) and (8), $i = n$.
($\Leftarrow$) Suppose $i = n$. Then $i-1 = n-1$.
By Lemma 2.1(iii), $\mathcal{D}_s(P^*_{n-1}, i-1) \neq \emptyset$.
Similarly, we prove $\mathcal{D}_s(P^*_{n-2}, i-2) \neq \emptyset$.
Since $i = n$, $i-2 = n-2 < n-1$.
By Lemma2.1(iv), $\mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) = \emptyset$.

iii) ($\Rightarrow$) Since $\mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) \neq \emptyset$, by Lemma 2.1(iv), $n-1 \leq i-2 \leq 2n-3$.
$\Rightarrow n+1 \leq i \leq 2n-1$

III. SECURE DOMINATING SETS AND SECURE DOMINATION POLYNOMIALS OF CENTIPEDES

In this section, we investigate secure dominating sets and secure domination polynomials of centipede.

$\Rightarrow n+\leq i$ (9)
Since $\mathcal{D}_s(P^*_{n-2}, i-2) \neq \emptyset$, by Lemma 2.1(iii),
$n-2 \leq i-2 \leq 2n-4$.
$\Rightarrow i-2 \leq 2n-4$
$\Rightarrow i \leq 2n-2$ (10)
From (9) and (10), $n+1 \leq i \leq 2n-2$.
($\Leftarrow$) Suppose $n+1 \leq i \leq 2n-2$.
Then $i \leq 2n-2$.
$\Rightarrow i-2 \leq 2n-4$
By Lemma 2.1(iii), $\mathcal{D}_s(P^*_{n-2}, i-2) \neq \emptyset$.
Since $n+1 \leq i$, $n-1 \leq i-2$. By Lemma 2.1(iv), $\mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) \neq \emptyset$.
Since $\mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) \neq \emptyset$ and $\mathcal{D}_s(P^*_{n-2}, i-2) \neq \emptyset$, by Lemma 2.2(iii),
$\mathcal{D}_s(P^*_{n-1}, i-1) \neq \emptyset$.

Theorem 2.4

i) If $\mathcal{D}_s(P^*_{n-1}, i-1) \neq \emptyset$, $\mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) \neq \emptyset$ and $\mathcal{D}_s(P^*_{n-2}, i-2) = \emptyset$, then

$$\mathcal{D}_s(P^*_n - \{2n\}, i) = \left\{ \begin{matrix} X \cup \{2n-1\} \\ /X \in \mathcal{D}_s(P^*_{n-1}, i-1) \end{matrix} \right\}$$

ii) If $\mathcal{D}_s(P^*_{n-1}, i-1) \neq \emptyset$, $\mathcal{D}_s(P^*_{n-2}, i-2) \neq \emptyset$ and $\mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) = \emptyset$, then

$$\mathcal{D}_s(P^*_n - \{2n\}, i) = \left\{ \begin{matrix} X \cup \{2n-3, 2n-2\}, \\ X \cup \{2n-3, 2n-1\}, \\ X \cup \{2n-2, 2n-1\} \\ /X \in \mathcal{D}_s(P^*_{n-2}, i-2) \end{matrix} \right\}$$

iii) If $\mathcal{D}_s(P^*_{n-1}, i-1) \neq \emptyset$, $\mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) \neq \emptyset$ and $\mathcal{D}_s(P^*_{n-2}, i-2) \neq \emptyset$, then

$$\mathcal{D}_s(P^*_n - \{2n\}, i) = \left\{ \begin{matrix} X_1 \cup \{2n-3, 2n-2\}, \\ X_1 \cup \{2n-3, 2n-1\}, \\ X_1 \cup \{2n-2, 2n-1\} \\ /X_1 \in \mathcal{D}_s(P^*_{n-2}, i-2) \end{matrix} \right\} \cup \left\{ \begin{matrix} X_2 \cup \{2n-2, 2n-1\} \\ /X_2 \in \mathcal{D}_s(P^*_{n-1} - \{2n-2\}, i-2) \end{matrix} \right\}$$

Theorem 2.5

For every $n \geq 3$,
$|\mathcal{D}_s(P^*_n - \{2n\}, i)| = |\mathcal{D}_s(P^*_{n-1}, i-1)| + |\mathcal{D}_s(P^*_{n-2}, i-2)|$.

Proof:

It follows from Theorem 2.4.

Here we state recursive formula for the secure domination polynomial of $P^*_n - \{2n\}$.

Theorem 2.6

For every $n \geq 3$,
$D_s(P^*_n - \{2n\}, x) = xD_s(P^*_{n-1}, x) + x^2 D_s(P^*_{n-2}, x)$.

Proof:

It follows from the definition of secure domination polynomial and Theorem 2.5.

Lemma 3.1

i) If $\mathcal{D}_s(P_{n-1}^*, i-1) \neq \emptyset$ and $\mathcal{D}_s(P_{n-1}^*, i-2) = \emptyset$, then $\mathcal{D}_s(P_n^* - \{2n\}, i-1) \neq \emptyset$.

ii) If $\mathcal{D}_s(P_{n-1}^*, i-1) = \emptyset$ and $\mathcal{D}_s(P_{n-1}^*, i-2) \neq \emptyset$, then $\mathcal{D}_s(P_n^* - \{2n\}, i-1) \neq \emptyset$.

iii) If $\mathcal{D}_s(P_{n-1}^*, i-1) \neq \emptyset$ and $\mathcal{D}_s(P_{n-1}^*, i-2) \neq \emptyset$, then $\mathcal{D}_s(P_n^* - \{2n\}, i-1) \neq \emptyset$.

Proof:

i) Since $\mathcal{D}_s(P_{n-1}^*, i-2) = \emptyset$, by Lemma 2.1 (iii),
$i-2 < n-1$ or $i-2 > 2n-2$.
$\Rightarrow i-1 < n$ or $i-1 > 2n-1$.
By Lemma 2.1(iv), $\mathcal{D}_s(P_n^* - \{2n\}, i-1) = \emptyset$.

ii) Since $\mathcal{D}_s(P_{n-1}^*, i-2) \neq \emptyset$, by Lemma 2.1(iii),
$n-1 \leq i-2 \leq 2n-2$.
$\Rightarrow n \leq i-1 \leq 2n-1$
By Lemma 2.1(iv), $\mathcal{D}_s(P_n^* - \{2n\}, i-1) \neq \emptyset$.

iii) Since $\mathcal{D}_s(P_{n-1}^*, i-2) \neq \emptyset$, by Lemma 2.1(iii) and (iv), $\mathcal{D}_s(P_n^* - \{2n\}, i-1) \neq \emptyset$.

Lemma 3.2

i) $\mathcal{D}_s(P_{n-1}^*, i-1) \neq \emptyset$, $\mathcal{D}_s(P_{n-1}^*, i-2) = \emptyset$ and $\mathcal{D}_s(P_n^* - \{2n\}, i-1) = \emptyset$ if and only if $i = n$.

ii) $\mathcal{D}_s(P_{n-1}^*, i-1) = \emptyset$, $\mathcal{D}_s(P_{n-1}^*, i-2) \neq \emptyset$ and $\mathcal{D}_s(P_n^* - \{2n\}, i-1) \neq \emptyset$ if and only if $i = 2n$.

iii) $\mathcal{D}_s(P_{n-1}^*, i-1) \neq \emptyset$, $\mathcal{D}_s(P_{n-1}^*, i-2) \neq \emptyset$ and $\mathcal{D}_s(P_n^* - \{2n\}, i-1) \neq \emptyset$ if and only if
$n+1 \leq i \leq 2n-1$.

Proof:

i) $(\Rightarrow)$ Since $\mathcal{D}_s(P_{n-1}^*, i-1) \neq \emptyset$ and $\mathcal{D}_s(P_n^* - \{2n\}, i-1) = \emptyset$ by Lemma 2.1(iii) and (iv), $n \leq i$ and $i < n+1$. We have $i = n$.
$(\Leftarrow)$ Suppose $i = n$.
Then $i-1 = n-1$. By Lemma 2.1(iii),
$\mathcal{D}_s(P_{n-1}^*, i-1) \neq \emptyset$ and $\mathcal{D}_s(P_n^* - \{2n\}, i-1) = \emptyset$.
Since $i = n$, $n-1 = i-1 > i-2$. By Lemma 2.1(iii), $\mathcal{D}_s(P_{n-1}^*, i-2) = \emptyset$.

ii) $(\Rightarrow)$ Since $\mathcal{D}_s(P_{n-1}^*, i-1) = \emptyset$, by Lemma 2.1(iii), $i-1 < n-1$ or $i-1 > 2n-2$.
$\Rightarrow i > 2n-1$ (11)
Since $\mathcal{D}_s(P_n^* - \{2n\}, i-1) \neq \emptyset$, by Lemma 2.1(iv), $n+1 \leq i-1 \leq 2n-1$.
$\Rightarrow i \leq 2n$ (12)
From (11) and (12), we have $i = 2n$.
$(\Leftarrow)$ Suppose $i = 2n$.
Then $i-1 = 2n-1$. By Lemma 2.1(iv),
$\mathcal{D}_s(P_n^* - \{2n\}, i-1) \neq \emptyset$.
Since $i = 2n$, $i-2 = 2n-2$. By Lemma 2.1(iii), $\mathcal{D}_s(P_{n-1}^*, i-2) \neq \emptyset$.
Since $i = 2n$, $i-1 = 2n-1 > 2n-2$. By Lemma 2.1(iii), $\mathcal{D}_s(P_{n-1}^*, i-1) = \emptyset$.

iii) $(\Rightarrow)$ Since $\mathcal{D}_s(P_{n-1}^*, i-1) \neq \emptyset$, by Lemma 2.1(iii), $n-1 \leq i-1 \leq 2n-2$.
$\Rightarrow i \leq 2n-1$ (13)
Since $\mathcal{D}_s(P_{n-1}^*, i-2) \neq \emptyset$, by Lemma 2.1(iv),
$n-1 \leq i-2 \leq 2n-2$.
$\Rightarrow n+1 \leq i$ (14)
From (13) and (14), we have $n+1 \leq i \leq 2n-1$.
$(\Leftarrow)$ Suppose $n+1 \leq i \leq 2n-1$.
Then $n \leq i-1$. By Lemma 2.1(iv),
$\mathcal{D}_s(P_n^* - \{2n\}, i-1) \neq \emptyset$.
Since $n+1 \leq i$, $n-1 \leq i-2$. By Lemma 2.1(iii), $\mathcal{D}_s(P_{n-1}^*, i-2) \neq \emptyset$.
Since $i \leq 2n-1$, $i-1 \leq 2n-2$. By Lemma 2.1(iii), $\mathcal{D}_s(P_{n-1}^*, i-1) \neq \emptyset$.

Theorem 3.3

i) If $\mathcal{D}_s(P_{n-1}^*, i-1) \neq \emptyset$, $\mathcal{D}_s(P_{n-1}^*, i-2) = \emptyset$ and $\mathcal{D}_s(P_n^* - \{2n\}, i-1) = \emptyset$, then

$$\mathcal{D}_s(P_n^*, i) = \left\{ \begin{matrix} X \cup \{2n-1\}, \\ X \cup \{2n\} \\ / X \in \mathcal{D}_s(P_{n-1}^*, i-1) \end{matrix} \right\}$$

ii) If $\mathcal{D}_s(P_{n-1}^*, i-1) = \emptyset$, $\mathcal{D}_s(P_{n-1}^*, i-2) \neq \emptyset$ and $\mathcal{D}_s(P_n^* - \{2n\}, i-1) = \emptyset$, then
$\mathcal{D}_s(P_n^*, i) = \{X \cup \{2n\} / X \in \mathcal{D}_s(P_n^* - \{2n\}, i-1)\}$.

iii) If $\mathcal{D}_s(P_{n-1}^*, i-1) \neq \emptyset$, $\mathcal{D}_s(P_{n-1}^*, i-2) \neq \emptyset$ and $\mathcal{D}_s(P_n^* - \{2n\}, i-1) \neq \emptyset$, then

$$\mathcal{D}_s(P_n^*, i) = \left\{ \begin{matrix} X_1 \cup \{2n-1\}, \\ X_1 \cup \{2n\} \\ / X_1 \in \mathcal{D}_s(P_{n-1}^*, i-1) \end{matrix} \right\} \cup \left\{ \begin{matrix} X_2 \cup \{2n\} \\ / X_2 \in \mathcal{D}_s(P_n^* - \{2n\}, i-1) \end{matrix} \right\}$$

i	1	2	3	4	5	6	7	8	9	10
$P_1^* - \{2\}$	1									
P_1^*	2	1								
$P_2^* - \{4\}$	0	3	1							
P_2^*	0	4	4	1						
$P_3^* - \{6\}$	0	0	6	5	1					
P_3^*	0	0	8	12	6	1				
$P_4^* - \{8\}$	0	0	0	12	16	7	1			
P_4^*	0	0	0	16	32	24	8	1		
$P_5^* - \{10\}$	0	0	0	0	24	44	30	9	1	
P_5^*	0	0	0	0	32	80	80	40	10	1

Table1: $d_s(P_n^*, i)$ and $d_s(P_n^* - \{2n\}, i)$

Theorem 3.4

For every $n \geq 3$,

$$|\mathcal{D}_s(P_n^*, i)| = |\mathcal{D}_s(P_n^* - \{2n\}, i-1)| + |\mathcal{D}_s(P_{n-1}^*, i-1)| + 2|\mathcal{D}_s(P_{n-1}^*, i-2)|$$

Proof:

It follows from Theorem 3.3.

Theorem 3.5

For every $n \geq 3$,

$$D_s(P_n^*, x) = xD_s(P_n^* - \{2n\}, x) + xD_s(P_{n-1}^*, x) + 2x^2 D_s(P_{n-2}^*, x)$$

Proof:

It follows from the definition of secure domination polynomial and Theorem 3.4.

Theorem 3.6

For every $n \geq 2$, $D_s(P_n^* - \{2n\}, x) = x^n(x+2)^{n-2}(x+3)$ and $D_s(P_n^*, x) = x^n(x+2)^n$.

Proof:

We shall prove both the equalities together by induction on n.

Since $D_s(P_2^* - \{2n\}, x) = x^2(x+2)^{2-2}(x+3) = x^2(x+3)$ and $D_s(P_2^*, x) = x^2(x+2)^2$. We have the result for $n = 2$.

Now, suppose the result are true for all-natural numbers less than n.

By Theorem 2.6 and induction hypothesis, we have:

$$\begin{aligned} D_s(P_n^* - \{2n\}, x) &= xD_s(P_{n-1}^*, x) + x^2 D_s(P_{n-2}^*, x) \\ &= x(x^{n-1}(x+2)^{n-1}) + x^2(x^{n-2}(x+2)^{n-2}) \\ &= x^n(x+2)^{n-2}(1+x+2) \\ &= x^n(x+2)^{n-2}(x+3) \end{aligned}$$

Now, by Theorem 3.5 and induction hypothesis, we have:

$$\begin{aligned} D_s(P_n^*, x) &= x\left(x^n(x+2)^{n-2}(x+3)\right) + x(x^{n-1}(x+2)^{n-1}) + 2x^2(x^{n-2}(x+2)^{n-2}) \\ &= x^n(x+2)^{n-2}(x(x+3) + (x+2) + 2) \\ &= x^n(x+2)^{n-2}(x^2+4x+4) \\ &= x^n(x+2)^{n-2}(x+2)^2 \\ &= x^n(x+2)^n \end{aligned}$$

Theorem 3.7

i) $d_s(P_n^*, i) = 2^{2n-i}\binom{n}{i-n}$, for every $n \in \mathbb{N}$ and $n \leq i \leq 2n$.

ii) $d_s(P_n^* - \{2n\}, i) = 2^{2n-i-2}(2\binom{n-1}{i-n} + \binom{n-2}{i-n})$, for $n \geq 2$ and $n \leq i \leq 2n-1$.

Proof:

i) By Theorem 3.6,

$$D_s(P_n^*, x) = x^n(x+2)^n = \sum_{k=0}^{n} \binom{n}{k} 2^{n-k} x^{n+k}$$

Thus, we have

$d_s(P_n^*, n+k) = 2^{n-k}\binom{n}{k}$, for $0 \leq k \leq n$.

Equivalently $d_s(P_n^*, i) = 2^{2n-i}\binom{n}{i-n}$, for $n \leq i \leq 2n$.

ii) By Theorem 2.5,

$$\begin{aligned} d_s(P_n^* - \{2n\}, i) &= d_s(P_{n-1}^*, i-1) + d_s(P_{n-2}^*, i-2) \\ &= 2^{2n-i-1}\binom{n-1}{i-n} + 2^{2n-i-2}\binom{n-2}{i-n} \\ &= 2^{2n-i-2}\left(2\binom{n-1}{i-n} + \binom{n-2}{i-n}\right) \end{aligned}$$

IV. CONCLUSION

This paper discusses and analyses the secure dominating sets of centipede and secure domination polynomials of centipede. Using recursive formula, we constructed the polynomial $D_s(P_n^*, x) = \sum_{i=n}^{2n} d_s(P_n^*, i)x^i$, which we call secure domination polynomial of P_n^* and obtain some properties of this polynomial.

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REFERENCES

[1] S.Alikhani and Y-H.Peng, "Dominating Sets and Domination Polynomials of Paths", *International Journal of Mathematics and Mathematical Sciences*, Vol.2009, 2009, Article ID 542040.

[2] S.Alikhani and Y-H.Peng, "Dominating Sets of centipedes", *Journal of Discrete Mathematical Sciences & Cryptography*, Vol.12 (2009), No.4, pp. 411-428.

[3] G. Chartrand and L. Lesniak (2005), Graphs & Digraphs, Fourth Edition, Champman \& Hall / CRC.

[4] E.J.Cockayne, P.J.P.Grobler, W.R.Grundlingh J.Munganga and J.H.Van Vuuren, "Protection of a graph", *Util, Math*., 67(2005),19-32.

[5] S.V.Divya Rashmi, S.Arumugam and Ibrahim Venkat, "Secure Domination in Graphs", *Int.J.Advance. Soft Comput. Appl.*, 8(2) (2016), 79-83.

[6] Frank Harary, Graph Theory, Addison-Wesley Publishing Company, Inc.

[7] J. Gross and J. Yellen, Handbook of Graph Theory, CRC Press(2004).

[8] T W Haynes, S T Hedetniemi and P.J Slater, "Fundamentals of Domination in Graphs", *Marcel Dekker*, New York, 1998.

[9] K. Lal Gipson , Subha T, "Secure Domination Polynomials Of Paths", *International Journal of Scientific & Technology Research*, Volume 9, Issue 2(Feb 2020), 6517-6521.

[10] A.Vijayan and K.Lal Gipson, "Dominating Sets and Domination Polynomials of Square of Paths", *Open Journal of Discrete Mathematics*,2013,3,60-69.

[11] A. Vijayan and K. Lal Gipson, "Dominating Sets of Square of Centipedes", *International Journal of Mathematical Archive*, 4(2), 2013, 298-310.

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