SQUARES OF SQUARE DIFFERENCE LABELING OF LAMP GRAPHS

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Abstract

Let $G(V,E)$ be a graph with p vertices and q edges. Let $f: V(G) \to \{0,1,2,\dots,p-1\}$ be a bijection. Define $f^*: E(G) \to N$ by $f^*_{ssd}(uv) = \left[(f(u))^2 - (f(v))^2\right]^2, \forall uv \in E(G)$. If f^*_{ssd} is injective then f^*_{ssd} is called squares of square difference labeling of G. A graph G which admits squares of square difference labeling is called squares of square difference graph. We applied squares of square difference labeling to different graphs Lamp graph L(G), Sequential graph SL(G), Alternative Sequential graph ALS(G)

1. Introduction

Every graph in this paper are simple finite, undirected and non-trivial graph $G = (V,E)$ with vertex set V and Edge set E. For graph theoretic terminology we refer to Harary. The square sum labeling is defined by V. Ajitha, S. Arumugam and K. A. Germina.

We have improved to the square of square difference and we investigated for different graphs like Lamp $L(S)$ graph, Sequence lamps $SL(G)$ graph, Alternative sequence lamp $ASL(S)$ graph, Fire Cracker $FC_n(G)$.

Definition 2.1

Lamp graph $L(G)$ is obtained by appending regular C_4 to the second vertices of regular C_5.

Definition 2.2

Sequence Lamp graph $SL(G)$ is obtained by appending the lamp graph sequentially.

Definition 2.3

Alternative Sequence Lamp graph $ASL(G)$ is obtained by appending the lamp graph with a path sequentially.

Theorem 3.1

The Lamp graph $L(G)$ is a squares of square difference graph.

Proof

Let $V = \{v_i / 1 \le i \le 5\} \cup \{u_i / 1 \le i \le 3\}$ be the vertex set of lamp graph.

Let $E = \{v_i v_{i+1}/1 \le i \le 4\} \cup \{v_5 v_1\} \cup \{u_i u_{i+1}/1 \le i \le 2\} \cup \{v_2 v_1\} \cup \{v_2 v_3\}$ be the edge set of lamp graph.

Here $|V(G)| = 8$ and $|E(G)| = 9$.

Define $f: V \to \{0, 1, 2, \ldots, 7\}$ by

$f(v_i) = \{i - 1/1 \le i \le 5\}$

$f(u_i) = \{5 + (i - 1)/1 \le i \le 3\}$

Clearly f is a bijection.

The induced edge labeling $f^*_{ssd}: E(G) \to N$ is defined as follows

$f^*_{ssd}(v_i v_{i+1}) = [[f(v_i)]^2 - [f(v_{i+1})]^2]^2 \ \forall 1 \le i \le 4$

$= [i^2 - 2i + 1 - i^2]^2$

$= [-2i + 1]^2 = 4i^2 + 1 - 4i.$

$f^*_{ssd}(v_5 v_1) = [[f(v_5)]^2 - [f(v_1)]^2]^2$

$= [[5 - 1]^2 - [1 - 1]^2]^2$

$= [4^2 - 0]^2 = [16]^2 = 256$

$f^*_{ssd}(u_i u_{i+1}) = [[f(u_i)]^2 - [f(u_{i+1})]^2]^2 \ \forall 1 \le i \le 2$

$= [[5 + (i - 1)]^2 - [5 + i - 1 + 1]^2]^2$

$= [(5 + i)^2 + 1 - 2(5 + i) - (5 + i)^2]^2$

$= [1 - 2(5) - 2(i)]^2 = [9 - 2i]^2$

$f^*_{ssd}(v_2 u_1) = [[f(v_2)]^2 - [f(u_1)]^2]^2$

$= [[2 - 1]^2 - [5 + 1 - 1]^2]^2$

$= [1^2 - (5)]^2 = [-24]^2$

$f^*_{ssd}(v_2 u_3) = [[f(v_2)]^2 - [f(v_3)]^2]^2$

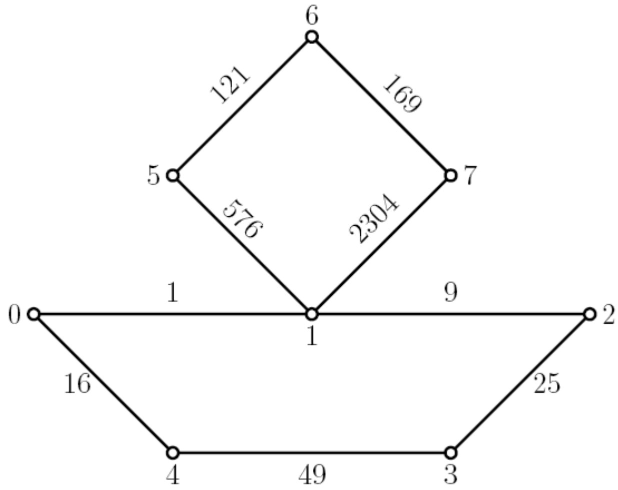
$= [[2 - 1]^2 - [5 + 3 - 1]^2]^2$

$= [1^2 - (5 + 2)]^2$

$= [1 - 49]^2 = [48]^2$

Clearly the edge labels are distinct.

Lamp graph is a square of square difference graph.



Theorem 3.2

The sequence lamp graph SL(G) is a squares of square difference graph.

Proof

Let $V = \{u_i/1 \leq i \leq 2n+1\} \cup \{v_i/1 \leq i \leq 2n\} \cup \{w_i/1 \leq i \leq 2n\} \cup \{z_i/1 \leq i \leq n\}$ be the vertex set of sequence lamps.

Let $E = \{u_iu_{i+1}/1 \leq i \leq 2n\} \cup \{u_iv_j/1 \leq i+2 \leq n; 1 \leq j+2 \leq n\} \cup \{u_iv_j/3 \leq i+2 \leq n; 2 \leq j+2 \leq n\} \cup \{u_iw_j, u_iw_{j+1}/2 \leq i+2 \leq n, 1 \leq j+2 \leq n\} \cup \{w_iz_j, w_{i+1}z_j/1 \leq i \leq 2n; 1 \leq j \leq n\} \cup \{v_iv_{i+1}/1 \leq i+2 \leq n\}$ be the edge set of sequence lamp graph.

Here $|V(G)| = 7n+1$ and $|E(G)| = 9n$.

Define $f: V \to \{0, 1, 2, \dots, p-1\}$ by

$f(u_i) = \{i - 1/1 \leq i \leq 2n+1\}$

$f(v_i) = \{2n + i)/1 \leq i \leq 2n\}$

$f(w_i) = \{4n + i)/1 \leq i \leq 2n\}$

$f(z) = \{6n + i)/1 \leq i \leq n\}$

Clearly f is a bijection.

The induced edge labeling $f^*_{ssd}: E(G) \to N$ is defined as follows

$$f^*_{ssd}(u_iu_{i+1}) = [[f(u_i)]^2 - [f(u_{i+1})]^2]^2 = [[i-1]^2 - [i+1-1]^2]^2$$
$$= [i^2 - 2i + 1 - i^2]^2 = [-2i+1]^2 = 4i^2 + 1 - 4i, \qquad \forall 1 \leq i \leq 2n.$$

$$f_{ssd}^*(u_iv_j) = \left[[f(u_i)]^2 - \left[f(v_j)\right]^2\right]^2 = [[i-1]^2 - [2n+j\,]^2]^2$$
$$= \ [i^2 - 2i + 1 - (4n^2 + j^2 + 4nj)]^2$$
$$= [i^2 - 2i + 1 - 4n^2 - j^2 - 4nj]^2, \forall\ 1 \le i+2 \le n; 1 \le j+2 \le n$$

$$f_{ssd}^*(u_iv_j) = \left[[f(u_i)]^2 - \left[f(v_j)\right]^2\right]^2 = [[i-1]^2 - [2n+j\,]^2]^2$$
$$= \ [i^2 - 2i + 1 - (4n^2 + j^2 + 4nj)]^2$$
$$= [i^2 - 2i + 1 - 4n^2 - j^2 - 4nj]^2, \forall\ 3 \le i+2 \le n; 2 \le j+2 \le n$$

$$f_{ssd}^*(u_iw_j) = \left[[f(u_i)]^2 - \left[f(w_j)\right]^2\right]^2 = [[i-1]^2 - [4n+j\,]^2]^2$$
$$= \ [i^2 - 2i + 1 - (16n^2 + j^2 + 8nj)]^2$$
$$= [i^2 - 2i + 1 - 16n^2 - j^2 - 8nj]^2, \forall\ 2 \le i+2 \le n; 1 \le j+2 \le n.$$

$$f_{ssd}^*(u_iw_{j+1}) = \left[[f(u_i)]^2 - \left[f(w_{j+1})\right]^2\right]^2 = [[i-1]^2 - [4n+j+1]^2]^2$$
$$= \ [i^2 - 2i + 1 - (16n^2 + j^2 + 1 + 8nj + 8n + 2j)]^2$$
$$= [i^2 - 2i + 1 - 16n^2 - j^2 - 1 - 8nj - 8n - 2j]^2$$
$$= [i^2 - 2i - 16n^2 - j^2 - 8nj - 8n - 2j]^2, \forall\ 2 \le i+2 \le n; 1 \le j+2 \le n.$$

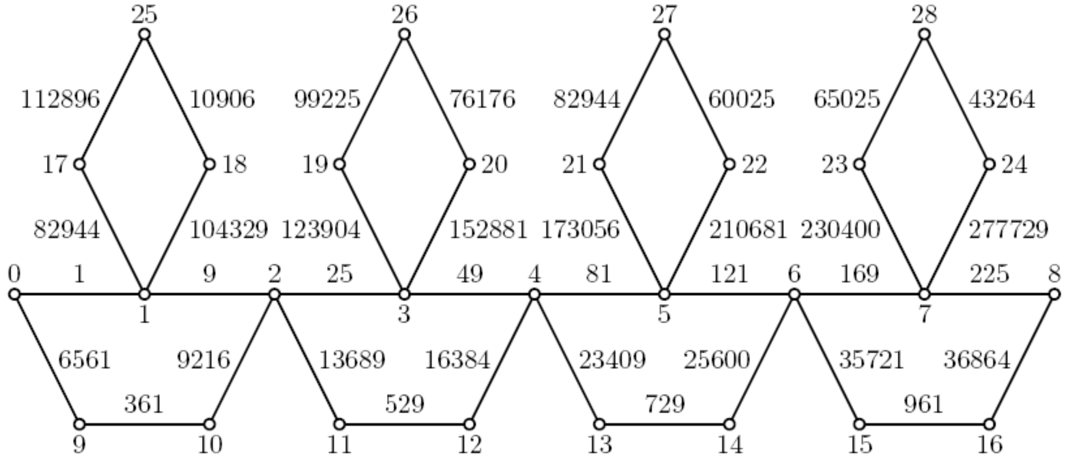
$$f_{ssd}^*(w_iz_j) = \left[[f(w_i)]^2 - \left[f(z_j)\right]^2\right]^2 = [[4n+i]^2 - [6n+j]^2]^2$$
$$= \ [16n^2 + i^2 + 8ni - (36n^2 + j^2 + 12nj)]^2$$
$$= [16n^2 + i^2 + 8ni - 36n^2 - j^2 - 12nj]^2$$
$$= [i^2 + 8ni - 20n^2 - j^2 - 12nj]^2, \forall\ 1 \le i \le 2n, 1 \le j \le n.$$

$$f_{ssd}^*(w_{i+1}z_j) = \left[[f(w_{i+1})]^2 - \left[f(z_j)\right]^2\right]^2 = [[4n+i+1]^2 - [6n+j]^2]^2$$
$$= \ [16n^2 + i^2 + 1 + 8ni + 8n + 2i - (36n^2 + j^2 + 12nj)]^2$$
$$= [16n^2 + i^2 + 1 + 8ni + 8n + 2i - 36n^2 - j^2 - 12nj]^2$$
$$= [i^2 + 1 + 8ni + 8n + 2i - 20n^2 - j^2 - 12nj]^2, \forall\ 1 \le i \le 2n, 1 \le j \le n.$$

$$f_{ssd}^*(v_iv_{i+1}) = [[f(v_i)]^2 - [f(v_{i+1})]^2]^2 = [[2n+i]^2 - [2n+i+1]^2]^2$$
$$= \ [4n^2 + i^2 + 4n - (4n^2 + i^2 + 1 + 4ni + 4n + 2i)]^2 = \ [1 - 4ni - 2i]^2,$$
$$\forall 1 \le i+2 \le n.$$

Clearly the edge labels are distinct.

Sequence lamps is a square of square difference graph.



Theorem 3.3

Alternative sequence lamp graph $ASL(G)$ is a squares of square difference graph .

Proof

Let $V = \{u_i/1 \leq i \leq 3n\} \cup \{v_i/1 \leq i \leq 2n\} \cup \{w_i/1 \leq i \leq 2n\} \cup \{z_i/1 \leq i \leq n\}$ be the vertex set of sequence lamps.

Let $E = \{u_i u_{i+1}/1 \leq i \leq 3n-1\} \cup \{u_i v_j/1 \leq i+3 \leq n; 1 \leq j+2 \leq n\} \cup \{u_i v_j/3 \leq i+3 \leq n; 2 \leq j+2 \leq n\} \cup \{u_i w_j, u_i w_{j+1}/2 \leq i+3 \leq n, 1 \leq j+2 \leq n\} \cup \{w_i z_j, w_{i+1} z_j/1 \leq i+2 \leq 2n; 1 \leq j \leq n\} \cup \{v_i v_{i+1}/1 \leq i+2 \leq 2n-1\}$ be the edge set of alternative sequence lamp graph.

Here $|V(G)| = 8n$ and $|E(G)| = 9n + 3$.

Define $f: V \to \{0, 1, 2, \ldots, p-1\}$ by

$f(u_i) = \{i - 1/1 \leq i \leq 3n\}$

$f(v_i) = \{3n + (i-1))/1 \leq i \leq 2n\}$

$f(w_i) = \{5n + (i-1))/1 \leq i \leq 2n\}$

$f(z) = \{7n + (i-1))/1 \leq i \leq n\}$

Clearly f is a bijection.

The induced edge labeling $f^*_{ssd}: E(G) \to N$ is defined as follows

$$\begin{aligned} f^*_{ssd}(u_i u_{i+1}) &= [[f(u_i)]^2 - [f(u_{i+1})]^2]^2 = [[i-1]^2 - [i+1-1]^2]^2 \\ &= [i^2 - 2i + 1 - i^2]^2 = [-2i+1]^2, \qquad 1 \leq i \leq 3n-1. \end{aligned}$$

$$\begin{aligned} f^*_{ssd}(u_i v_j) &= \left[[f(u_i)]^2 - \left[f(v_j)\right]^2\right]^2 = [[i-1]^2 - [3n+j-1]^2]^2 \\ &= [i^2 - 2i + 1 - (9n^2 + j^2 + 1 + 6nj - 6n - 2j)]^2 \\ &= [i^2 - 2i + 1 - 9n^2 - j^2 - 1 - 6nj + 6n + 2j]^2 \\ &= [i^2 - 2i - 9n^2 - j^2 - 6nj + 6n + 2j]^2 \ \forall\ 1 \leq i+3 \leq n; 1 \leq j+2 \leq n \end{aligned}$$

$$f_{ssd}^{*}(u_i v_j) = \left[[f(u_i)]^2 - \left[f(v_j)\right]^2\right]^2 = [[i-1]^2 - [3n+j-1]^2]^2$$
$$= [i^2 - 2i + 1 - (9n^2 + j^2 + 1 + 6nj - 6n - 2j)]^2$$
$$= [i^2 - 2i + 1 - 9n^2 - j^2 - 1 - 6nj + 6n + 2j]^2$$
$$= [i^2 - 2i - 9n^2 - j^2 - 6nj + 6n + 2j]^2 \ \forall\ 3 \le i+3 \le n; 2 \le j+2 \le n$$

$$f_{ssd}^{*}(u_i w_j) = \left[[f(u_i)]^2 - \left[f(w_j)\right]^2\right]^2 = [[i-1]^2 - [5n+j-1\ ]^2]^2$$
$$= [i^2 - 2i + 1 - (25n^2 + j^2 + 1 + 10nj - 10n - 2j)]^2$$
$$= [i^2 - 2i + 1 - 25n^2 - j^2 - 1 - 10nj + 10n + 2j]^2$$
$$= [i^2 - 2i - 25n^2 - j^2 - 10nj + 10n + 2j]^2, \forall\ 2 \le i+3 \le n; 1 \le j+ \le n.$$

$$f_{ssd}^{*}(u_i w_{j+1}) = \left[[f(u_i)]^2 - \left[f(w_{j+1})\right]^2\right]^2 = [[i-1]^2 - [5n+j-1+1]^2]^2$$
$$= [[i-1]^2 - [5n+j]^2]^2 = [i^2 - 2i - 25n^2 - j^2 - 10nj + 1]^2, \forall\ 2 \le i+3 \le n; 1 \le j+2 \le n.$$

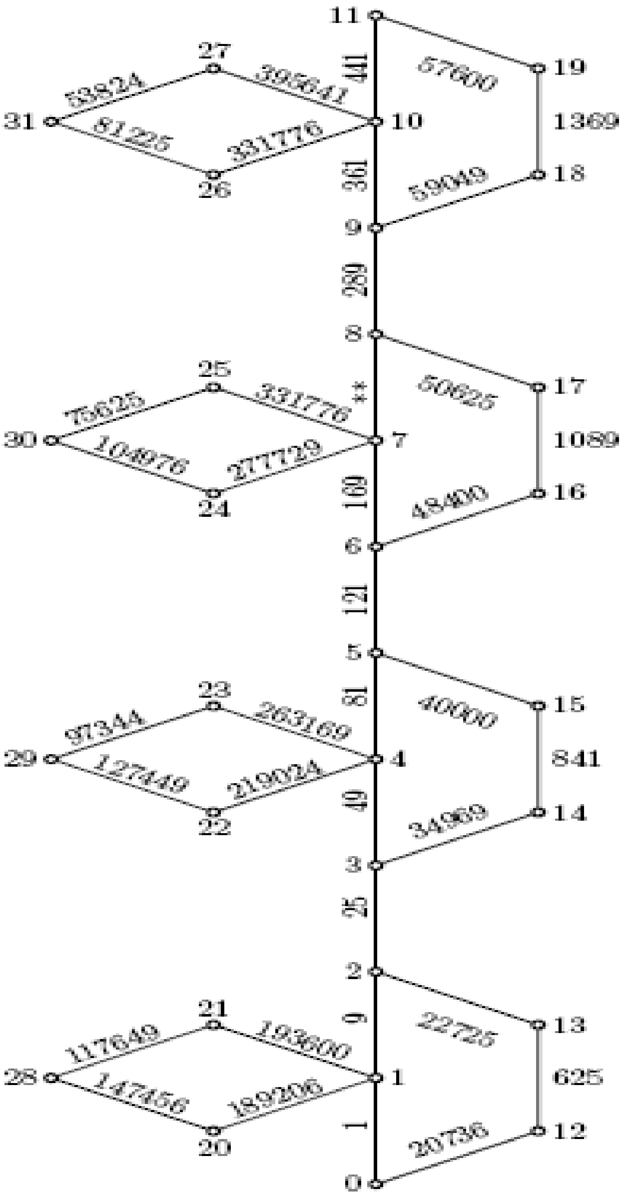
$$f_{ssd}^{*}(w_i z_j) = \left[[f(w_i)]^2 - \left[f(z_j)\right]^2\right]^2 = [[5n+(i-1)]^2 - [7n+j-1]^2]^2$$
$$= [[5n+i-1]^2 - [7n+j-1]^2]^2$$
$$= [25n^2 + i^2 + 10ni - 10n - 2i - (49n^2 + j^2 + 1 + 14nj - 14n - 2j)]^2$$
$$= [25n^2 + i^2 + 10ni - 10n - 2i - 49n^2 - j^2 - 1 - 14nj + 14n + 2j]^2$$
$$= [-24n^2 + i^2 - j^2 + 10ni - 14nj + 4n - 2i + 2j]^2, \forall\ 1 \le i+2 \le 2n, 1 \le j \le n.$$

$$f_{ssd}^{*}(w_{i+1} z_j) = \left[[f(w_{i+1})]^2 - \left[f(z_j)\right]^2\right]^2 = [[5n+i-1+1]^2 - [7n+j-1]^2]^2$$
$$= [25n^2 + i^2 + 10ni - (49n^2 + j^2 + 1 + 14nj - 14n - 2j)]^2$$
$$= [-24n^2 + i^2 + 10ni - j^2 - 1 - 14nj + 14n + 2j]^2 \ \forall\ 1 \le i+2 \le 2n, 1 \le j \le n.$$

$$f_{ssd}^{*}(v_i v_{i+1}) = [[f(v_i)]^2 - [f(v_{i+1})]^2]^2 = [[3n+(i-1)]^2 - [3n+i-1+1]^2]^2$$
$$= [9n^2 + i^2 + 1 + 6ni - 6n - 2i - (3n+i)^2]^2$$
$$= [9n^2 + i^2 + 1 + 6ni - 6n - 2i - (9n^2 + i^2 + 6ni)]^2 = [1 - 6n - 2i]^2, \forall 1 \le i+2 \le n.$$

Clearly the edge labels are distinct.

Alternative Sequence lamps is a square of square difference graph.



4 Conclusions

In this paper, we proved Lamp graph L(G),Sequence of Lamp graph SL(G), Alternative Sequence of Lamp graph ASL(G) are squares of square difference graphs. There may be interesting squares of square difference graphs can be constructed in future.

References

[1] A Galian, A dyanamic survey of graph labeling, The electronic journal of combinatories (2019).

[2] A Galian, A dyanamic survey of graph labeling, The electronic journal of combinatories (2019).

[3] Germina K.A, Reena Sebastian, On Square Sum Graphs,Proyecciones Jouranal of Mathematics, Vol. 32 No. 2, pp. 107-117 June 2013.

[4] J. Shiama, *Square Difference Labeling for Some Graphs*, InternationalJournal of Computer Application (0975- 8887) Vol. 44- No.4 April 2012

[5] Sunoj B.S. Mathew Varkey T.K, *Square difference prime labeling for some snake graphs,* Global Journal of Pure and Applied Mathematics, ISSN: 0973-1768, Vol. 13, No. 3(2017), p.p 108-1089.

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