Comparative study of Predication evolution of ARMA, AR-GARCH, and ARMA-GARCH: Sunspot Cycles as a case study

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Abstract:

This study focuses on the appropriateness of forecasting evolution of various Autoregressive and moving average models and generalized autoregressive conditional heteroskedasticity (GARCH) models in terms of their performance for delivering volatility forecasts for Sunspot cycles. It is verified that the sunspot cycles have stationary nature with second difference and Autocorrelation (AC), Partial Autocorrelation (PAC) and Ljung-Box Q-statistics test are used to check the presence of white noise (strongly correlated) in the time series data. Under model Identification and Estimation, diagnostic checking and Forecasting the ARMA, AR (p)-GARCH and ARMA-GARCH models are found to be most appropriate. To detect the appropriateness of Autoregressive Conditional Heteroscedastic (ARCH) effect on sunspot cycles data, Lagrange Multiplier test is used. The selection of the model is based on residual diagnostic checking's such as ARCH LM, normality test and correlogram squared residuals. Forecasting evolutions are verified by Root Mean Squared Error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE) and Theil's U-Statistics test (U test). Least square Estimation is used to investigate the ARMA process and the Gaussian quasi maximum likelihood estimation (QMLE) is used to estimate GARCH (1, 1) process for the specification of AR (p) and ARMA (p, q) models. The selection of models is based on the least value of Durbin-Watson state. The novelty of this study to forecasts for the evolution of sunspot cycles obtained by ARMA, AR-GARCH and ARMA-GARCH models are compared. RMSE, MAE and U test is utilized to check the appropriateness of various ARMA models. Only the MAPE exhibited the appropriateness of ARMA-GARCH model apart from cycles 1^{st}, 9^{th}, 10^{th} and 17^{th} follows ARMA models and cycles 2^{nd}, 3^{rd}, 4^{th}, 5^{th}, 6^{th} and 20^{th} follows AR-GARCH model.

Keywords: ARMA, AR-GARCH, ARMA-GARCH, Durbin-Watson, Theil's U-Statistics

1: Introduction

A strong magnetic field exists in the outer region of the sun which is demonstrated by temporal dynamics and complex. For instance, sunspots, solar flares, and solar wind velocity. Sunspots have a strong statistical relationship with chromospheric flares are found (Bray and Laughhead, 1997). The main cause for change in the long term variation of the earth's climate is solar activity. Whereas, short term variation brings fluctuation in meteorological parameters (Wittmann, 1978). Sunspot cycles have a strong periodic pattern in which the number of sunspots increase and decrease within 11 years. In the sunspot data has various long and short term periods presented, apart from the prominent 11-year period. These are known as "G – O" rule or the "odd – even" rule. This rule describes that an odd number of cycles are stronger as compared to the preceding even number one. Time series are very useful in various disciplines of solar physics, climate change and so on. Many time series techniques are used to predict sunspot cycles like ARMA models, ARIMA models, GRACH models, Curve fitting and so on. These methods bring more appropriate forecasting of sunspot cycles. The Autoregressive models can involve both stationary and non-stationary time series. A statistical approach to forecasting involves stochastic models to predict the values of sunspot cycles by using the previous once. In the linear time series, two methods are frequently used in literature, viz. Autoregressive AR (p) and Moving Average MA (q) (Jenkins et.al. 1970 and Hipal et. al. 1994).

2: Materials and Methods

Basically, this study is investigating the mean monthly sunspot cycles from (1755 to 2019), which is consisted (1^{st} to 24^{th}) cycles. The World Data Centre (WDC) and the National Oceanic and Atmospheric Administration (NOAA) for providing the 24^{th} Sunspot cycle data. The Statistical EViews version 9.0 software is used for calculation and analysis of ARMA (p, q), AR (p) – GRACH (1, 1) and ARMA (p, q) -GRACH (1, 1) model and respective graphs, for instance (Time series plots and fitted, residual and predicted plots for total sunspot cycles are most helpful for sunspot cycles scrutiny). Section 2 consists of two subsections.

The sunspot cycles can be estimated and forecasted by various statistical methods. This research mainly focuses on the Box-Jenkins method for the stationary process of ARMA (p, q), AR (p) -GRACH (1, 1) and ARMA (p, q) -GRACH (1, 1). The forecasting ability of each model of sunspot cycles will be judged by diagnostic checking tests such as Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE). Mean maximum likelihood estimation is used to analyze ARMA (p, q), AR (p) -GRACH (1, 1) and ARMA (p, q) -GRACH (1, 1). Selection for the appropriate and adequate process of ARMA (p, q), AR (p) – GARCH (1, 1) and ARMA (p, q) – GARCH (1, 1) for the accurate and significant test are very important. Durbin-Watson (DW) statistics test, Akaike information criterion (AIC), Bayesian Schwarz information criterion (SIC) and Hannan Quinn information criterion (HQC) and Log maximum likelihood test are used to select appropriate models. The selection of appropriate sunspot cycles is based on the minimum value of the Durbin-Watson (DW) statistics test for each method.

ARMA models are developed by (Jenkins et. al 1994). An ARMA model is the combination of an idea of Autoregressive AR (p) and Moving Average MA (q) process. The concept of ARMA process is strongly relevant in volatility modeling. ARMA model is widely used for forecasting future values. Autoregressive process (AR) is developed by (yule, 1927). In the stochastic process, Autoregressive process AR (p) can be expressed by a weighted sum of its previous value and white noise. The generalized Autoregressive process AR (p) of lag p as follow

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} + \epsilon_t \quad (1)$$

Here ε_t is white noise with mean E (ϵ_t) = 0, variance Var (ϵ_t) = σ^2 and Cov (ϵ_{t-s}, ϵ_t) = 0, if $s \neq 0$. For every t, suppose that τ_t is independent of the $X_{t-1}, X_{t-2}, \ldots$. τ_t is uncorrelated with X_s for each $s < t$. AR (p) models regress is past values of the data set. Whereas, MA (q) model relates with error terms as a descriptive variable (Hipal et. al. 1994). The generalized Moving Average process MA (q) of lag q as follows

$$X_t = \epsilon_t + \beta_1 \epsilon_{t-1} + \beta_2 \epsilon_{t-2} + \ldots + \beta_q \epsilon_{t-q} \quad (2)$$

The process X_t is defined by the ARMA model

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} + \epsilon_t + \beta_1 \epsilon_{t-1} + \beta_2 \epsilon_{t-2} + \ldots + \beta_q \epsilon_{t-q} \quad (3)$$

With ϵ_t is an uncorrelated process with mean zero. The prediction of ARMA (p, q) process shows the decay to be sinusoidally and exponentially to zero.

Autoregressive process (AR) is developed by Yule (Yule, 1927). An autoregressive process AR (p) can be expressed by a weighted sum of its previous values and a white noise. The generalized Autoregressive process AR (p) of lag p is expressed as follows.

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} + \tau_t \quad (4)$$

Here ε_t is white noise with mean E (τ_t) = 0, variance Var (τ_t) = σ^2 and Cov (τ_{t-s}, τ_t) = 0, if $s \neq 0$. For every t, it is supposed that τ_t is independent of the $X_{t-1}, X_{t-2}, \ldots$. τ_t is uncorrelated with X_s for each $s < t$.

The GARCH (1, 1) process was developed by Bollerslev (1986). The generalized GARCH (p, q) stochastic volatility model was developed by (Aquilar et. al. 2000, Kim et. al. 1998). The generalized autoregressive conditional heteroskedasticity GARCH model is more appropriate to analyze the fluctuation of variances. GARCH model is preferred to study volatility clustering () and the relative volatility forecasts (Andersen et. al. 1998). GARCH (1, 1) process has better forecasting ability as compared to other traditional models (Akgiray, 1989). GARCH (1, 1) process is better to construct multi-period long term forecasting. GARCH model is widely used for modeling and forecasting of various other types of data which include economic and financial modeling too. The GARCH (1, 1) process of X_t is represented as follows

$$\tau_t = \sigma_t \epsilon_t \quad (5)$$

Where $\epsilon_t \sim IID(0, 1)$

The model itself is represented as

$$\sigma_t^2 = \delta + \beta \, \tau_{t-1}^2 + \gamma \, \sigma_{t-1}^2 \text{ with } \delta + \beta + \gamma \geq 0$$

GARCH (1, 1) process is a covariance-stationary white noise process if and only if $\beta + \gamma < 1$. The variance of the covariance-stationary process is represented as follows.

$$\text{Var}(X_t) = \frac{\delta}{1-\beta-\gamma} \quad (6)$$

GARCH (1, 1) process is stationary if it holds stationary conditions. GARCH (1, 1) process mostly follows leptokurtic (kurtosis greater than 3) which shows a heavy tailed behavior (long term dependence). GARCH (1, 1) process specification with AR (p) is defined as follows ($t = 0 \pm 1, \pm 2, \ldots$).

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} + \tau_t \quad (7)$$

$$\tau_t = \sigma_t \epsilon_t \text{ with } \in_t \sim IID(0, 1) \quad (8)$$

$$\sigma_t^2 = \delta + \beta_1 \tau_{t-1}^2 + \beta_2 \tau_{t-2}^2 + \cdots + \beta_p \tau_{t-p}^2 + \gamma \, \sigma_{t-1}^2 \quad (9)$$

Where $E(\tau_t) = 0$, variance $\text{Var}(\tau_t | \tau_{t-1}^2, \tau_{t-2}^2 \ldots) = \sigma^2$ and $\text{Cov}(\tau_{t-s}, \tau_t) = 0$, if $s \neq 0$.

Moreover, The Box-Jenkins methodology with GARCH approach is used to develop models, to estimate the models and to forecast the sunspot cycle's data.

The concept of ARMA models is strongly relevant to volatility modeling. The generalized autoregressive conditional heteroscedastic (GARCH) models can be linked as ARMA models. GARCH Models satisfy an ARMA equation with white noise. In the time series, the GARCH model supposition that conditional mean is zero. Generally, the conditional mean of ARMA model can be structured. Identification of GARCH process focused on the square of residuals from the appropriate ARMA models. Moreover, in the ARAM process, the quasi maximum likelihood estimation is nearly independent of their GARCH process. ARMA estimates and GARCH estimation are strongly correlated if the ARMA – GARCH process has a skewed distribution (Csyer et. al 2008). The ARAM process and GARCH process have similar behavior in forecasting. ARMA – GARCH process provides a good estimation in time series data.

GARCH (1, 1) process specification with ARMA (p, q) is defined as follows ($t = 0 \pm 1, \pm 2, \ldots$).

$$X_t = \alpha_1 X_{t-1} + \alpha_2 X_{t-2} + \ldots + \alpha_p X_{t-p} + \in_t + \beta_1 \in_{t-1} + \beta_2 \in_{t-2} + \ldots + \beta_q \in_{t-q} \quad (10)$$

$$\tau_t = \sigma_t \epsilon_t \text{ with } \in_t \sim IID(0, 1) \quad (11)$$

$$\sigma_t^2 = \delta + \beta_1 \tau_{t-1}^2 + \beta_2 \tau_{t-2}^2 + \cdots + \beta_p \tau_{t-p}^2 + \gamma \, \sigma_{t-1}^2 \quad (12)$$

Where $E(\tau_t) = 0$, variance $\text{Var}(\tau_t | \tau_{t-1}^2, \tau_{t-2}^2 \ldots) = \sigma^2$ and $\text{Cov}(\tau_{t-s}, \tau_t) = 0$, if $s \neq 0$.

Moreover, The Box-Jenkins methodology with GARCH approach is used to develop models, to estimate the models and to forecast the sunspot cycle's data.

Lagrange multipliers (LM) are used to check the ARCH effect of the available data. Correlogram squared residual test is also used to verify the ARCH effect. The Gaussian quasi maximum likelihood estimation is performed to select the appropriate model. Forecasts with the best fitted model of sunspot cycles were tested for accuracy with the help of Root mean square error (RMSE), Mean Absolute Error (MAE) and Mean Absolute Percentage Error (MAPE). A little description of these terminologies is given in the following.

$$\text{MAE} = \frac{1}{n}\sum_{t=1}^{n}|\epsilon_t| \tag{13}$$

Where n is the number of observations. Mean Absolute Error (MAE) measures the absolute deviation of forecasted values from real ones. It is also known as Mean Absolute Deviation (MAD). It expresses the magnitude of overall error, happened due to forecasting. MAE does not cancel out the effect of positive and negative errors. MAE does not express the directions of errors. It should be as small as possible for good forecasting. MAE depends on the data transformations and the scale of measurement. MAE is not the panelized the extreme forecast error.

The Mean Absolute Percentage Error (MAPE) is defined as

$$\text{MAPE} = \frac{1}{n}\sum_{t=1}^{n}\left|\frac{\epsilon_t}{X_t}\right| \times 100 \tag{14}$$

Mean Absolute Percentage Error (MAPE) provides the percentage of the average absolute error. It is independent of the scale measurement. Whereas, dependent on the data transformations. MAPE does not locate the direction of Error. The extreme deviation is not penalized by MAPE. Opposite signed errors do not parallel to each other in MAPE.

The root mean squared error (RMSE) is defined as

$$\text{RMSE} = \sqrt{\frac{1}{n}\sum_{t=1}^{n}\epsilon_t^2} \tag{15}$$

RMSE calculate the average squared deviation of forecasted values. The opposed sighed error does not parallel to each other. RMSE provide the complete idea of the error happened during forecasting. While forecasting, penalizes extreme error happens. In RMSE, the total forecast error is affected by the large individual error. For instance, a large error is more appropriate than small errors. It does not reveal the direction of overall errors. RMSE is affected by the data transformation and change of scale. RMSE is a good measure of overall forecast error.

3: Results and Discussion

In this study, forecasting accuracy of ARMA, AR-GARCH (1, 1) and ARMA-GARCH (1, 1) models are checked by RMSE, MAE, MAPE and Theil's U-Statistics (U test).

Duration	ARMA (p, q)		AR(p)–GARCH (1,1)		ARMA(p,q))–GARCH (1,1)	
Aug 1755 - Mar 1766	(6,5)	22.96202	AR(2)	27.61330	(6,5)	32.67503
Mar 1766 - Aug 1775	(5,3)	37.92649	AR(2)	48.9534	(5,1)	43.75567
Aug 1775 - Jun 1784	(6,4)	56.72805	AR(2)	81.25105	(1,1)	82.97762
Jun 1784 - Jun 1798	(5,4)	54.15024	AR(2)	69.70664	(2,2)	69.78911
Jun 1798 - Sep 1810	(5,3)	19.61195	AR(2)	29.84910	(3,3)	28.93611
Sep1810 - Dec 1823	(3,4)	17.58213	AR(2)	23.87355	(3,3)	25.17780
Dec1823 - Oct 1833	(5,3)	24.84239	AR(3)	26.97423	(3,3)	35.06270
Oct1833 - Sep 1843	(4,6)	47.89669	AR(2)	76.51305	(3,2)	70.49102
Sep1843 - Mar 1855	(4,3)	36.85102	AR(2)	54.20801	(4,2)	55.68955
Mar1855 - Feb 1867	(6,4)	34.05871	AR(2)	46.84207	(4,4)	50.33423
Feb1867 - Sep 1878	(6,4)	51.34360	AR(2)	66.48149	(5,1)	68.21752
Sep1878 - Jun 1890	(5,3)	26.94070	AR(2)	39.33113	(2,2)	40.16201
Jun1890 - Sep 1902	(5,4)	33.19800	AR(2)	41.43307	(2,2)	44.99864
Sep 1902 - Dec 1913	(4,6)	26.33349	AR(2)	37.13987	(2,2)	36.86236
Dec1913 - May 1923	(5,4)	33.40646	AR(3)	50.69620	(3,3)	53.34212
May 1923 - Sep 1933	(6,5)	28.41330	AR(2)	41.61186	(2,2)	44.92782
Sep 1933 - Jan 1944	(6,2)	40.50563	AR(3)	66.90030	(2,2)	67.16046
Jan 1944 - Feb 1954	(6,4)	58.17556	AR(2)	88.43862	(5,3)	89.57113
Feb 1954 - Oct 1964	(4,3)	81.95903	AR(2)	110.3128	(5,3)	113.2087
Oct 1964 - May 1976	(4,3)	39.00213	AR(2)	59.36716	(2,2)	63.56454
May1976 - Mar 1986	(5,3)	61.20026	AR(2)	92.03096	(2,2)	91.74100
Mar1986 - Jun1996	(5,6)	65.05685	AR(2)	84.07413	(6,1)	87.08569
Jun1996 - Jan 2008	(6,4)	45.06462	AR(2)	60.54590	(2,2)	65.31769
Jan 2008 - Dec 2019	(6,4)	44.54175	AR(2)	52.01005	(2,2)	54.37098

Table 1: Root Mean Square Error (RMSE) of ARMA, AR-GARCH and ARMA-GARCH of sunspot cycles

Table 1: depicts that the comparative study of the forecasting evolution of ARMA (p, q), AR (p)-GARCH (1, 1) and ARMA (p, q)-GARCH (1, 1) process of each sunspot cycle in term of RMSE. 1st Sunspot cycles has best fitted models are ARMA (6, 5), AR (2)-GARCH and ARMA (6, 5)-GARCH process. 2nd Sunspot cycles has expressed that the appropriate models are ARMA (5, 3), AR (2)-GARCH and ARMA (5, 1)-GARCH process. 3rd sunspot cycle has suitable fitted models are ARMA (6, 4), AR (2) -GARCH and ARMA (1, 1) –GARCH process. 4th sunspot cycle has best fitted models are ARMA (5, 4), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 5th sunspot cycle shows appropriate models are ARMA (5, 3), AR (2)-GARCH and ARMA (3, 3)-GARCH process. 6th sunspot cycles express the suitable models are ARMA (3, 4), AR (2)-GARCH and ARMA (3, 3)-GARCH process. 7th sunspot cycles depict best fitted models are ARMA (5, 3), AR (3)-GARCH and ARMA (3, 3)-GARCH process. 8th sunspot cycle shows best appropriate models are ARMA (4, 6), AR (2)-GARCH and ARMA (3, 2)-GARCH process. 9th sunspot cycle has best fitted models are ARMA (4, 3), AR (2)-GARCH and ARMA (4, 2)-GARCH process. 10th sunspot cycles express best fitted models are ARMA (6, 4), AR (2)-GARCH and ARMA (4, 4)-GARCH process. 11th sunspot cycles reveal best fitted models are ARMA (6, 4), AR (2)-GARCH and ARMA (5, 1)-GARCH process. 12th sunspot cycle has suitable models are ARMA (5, 3), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 13th sunspot cycle shows best fitted models are

ARMA (5, 4), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 14th sunspot cycles express appropriate models are ARMA (4, 6), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 15th sunspot cycles describe the best fitted models are ARMA (5, 4), AR (3)-GARCH and ARMA (3, 3)-GARCH process. 16th sunspot cycles represent suitable models are ARMA (6, 5), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 17th sunspot cycle has best fitted models are ARMA (6, 2), AR (3)-GARCH and ARMA (2, 2)-GARCH process. 18th sunspot cycles describe the appropriate models are ARMA (6, 4), AR (2)-GARCH and ARMA (5, 3)-GARCH process. 19th sunspot cycle has best fitted models are ARMA (4, 3), AR (2)-GARCH and ARMA (5, 3)-GARCH process. 20th sunspot cycles reveal best fitted models are ARMA (4, 3), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 21st sunspot cycles express suitable models are ARMA (5, 3), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 22nd sunspot cycle has best fitted models are ARMA (5, 6), AR (2)-GARCH and ARMA (6, 1)-GARCH process. 23rd sunspot cycles describe appropriate models are ARMA (6, 4), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 24th sunspot cycles represent suitable models are ARMA (6, 4), AR (2)-GARCH and ARMA (2, 2)-GARCH process. Root Mean Square Error (RMSE) shows that ARMA (p, q) model is the best appropriate model as comparative to AR(p)-GARCH and ARMA (p, q)- GARCH models. Sunspot cycles have least RMSE values in ARMA (p, q) process. Figure 1 displayed that RMSE comparison of ARMA, AR-GARCH and ARMA-GARCH process of sunspot cycles.

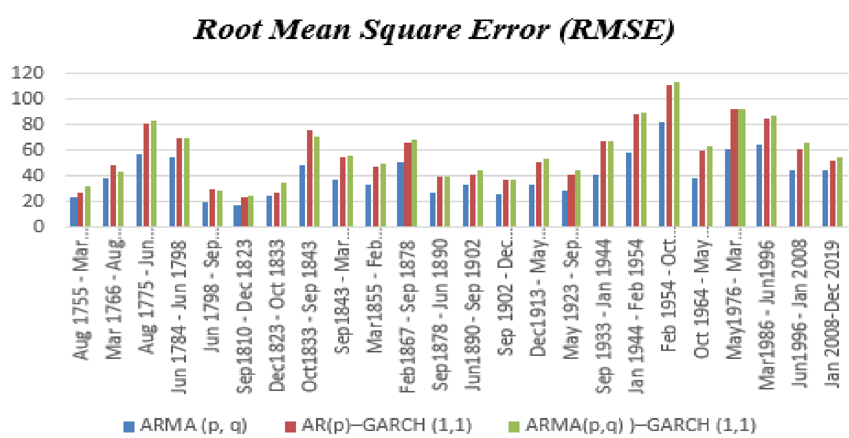

Figure 1: RMSE of ARMA, AR-GARCH and ARMA-GARCH process of sunspot cycles.

The selection of the best fitted model is based on MAE.

Duration	ARMA (p, q)		AR(p)–GARCH (1,1)		ARMA(p,q))–GARCH (1,1)	
Aug 1755 - Mar 1766	(4,5)	18.43576	AR(2)	21.4119	(6,5)	25.67541
Mar 1766 - Aug 1775	(5,3)	29.55789	AR(2)	37.4807	(5,1)	33.31524
Aug 1775 - Jun 1784	(6,4)	43.64520	AR(2)	62.90952	(1,1)	64.99252
Jun 1784 - Jun 1798	(5,6)	41.59299	AR(2)	53.90236	(2,2)	53.91451
Jun 1798 - Sep 1810	(5,3)	16.77907	AR(2)	23.36523	(3,3)	22.58619
Sep1810 - Dec 1823	(6,4)	13.25282	AR(2)	16.61689	(3,3)	22. 61958

Dec1823 - Oct 1833	(5,3)	20.57404	AR(3)	22.29147	(3,3)	27.96897
Oct1833 - Sep 1843	(4,6)	37.64070	AR(2)	59.98412	(3,2)	53.86759
Sep1843 - Mar 1855	(6,4)	26.67266	AR(2)	41.01302	(4,2)	42.46238
Mar1855 - Feb 1867	(6,4)	26.32748	AR(2)	36.86088	(4,4)	40.31416
Feb1867 - Sep 1878	(6,4)	40.82203	AR(2)	48.35055	(5,1)	50.46720
Sep1878 - Jun 1890	(5,3)	22.33445	AR(2)	29.41939	(2,2)	30.27203
Jun1890 - Sep 1902	(5,4)	27.39173	AR(2)	30.79801	(2,2)	33.86295
Sep 1902 - Dec 1913	(4,6)	21.89815	AR(2)	28.22192	(2,2)	27.61363
Dec1913 - May 1923	(5,3)	26.04595	AR(3)	39.37540	(3,3)	41.91097
May 1923 - Sep 1933	(6,5)	23.46526	AR(2)	32.01947	(2,2)	35.31193
Sep 1933 - Jan 1944	(6,2)	31.48136	AR(3)	32.01947	(2,2)	54.07226
Jan 1944 - Feb 1954	(5,3)	44.02761	AR(2)	70.49887	(5,3)	72.19864
Feb 1954 - Oct 1964	(4,3)	64.00049	AR(2)	84.95945	(5,3)	88.62173
Oct 1964 - May 1976	(4,3)	32.19808	AR(2)	47.77713	(2,2)	51.95468
May1976 - Mar 1986	(5,3)	50.64551	AR(2)	73.19906	(2,2)	72.87236
Mar1986 - Jun1996	(4,5)	50.90771	AR(2)	63.14946	(6,1)	66.45792
Jun1996 - Jan 2008	(3,4)	35.46916	AR(2)	46.23687	(2,2)	50.61469
Jan 2008-Dec 2019	(6,4)	35.30753	AR(2)	37.04478	(2,2)	38.55961

Table 2: Mean Absolute Error (MAE) of ARMA, AR-GARCH and ARMA-GARCH of sunspot cycles

Table 2: depict that 1st sunspot cycles has best fitted models are ARMA (4, 5), AR (2)-GARCH and ARMA (6, 5)-GARCH process. 2nd sunspot cycle has expressed that the appropriate models are ARMA (5, 3), AR (2)-GARCH and ARMA (5, 1)-GARCH process. 3rd sunspot cycle has suitable fitted models are ARMA (6, 4), AR (2) -GARCH and ARMA (1, 1) -GARCH process. 4th sunspot cycle has best fitted models are ARMA (5, 6), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 5th sunspot cycle shows appropriate models are ARMA (5, 3), AR (2)-GARCH and ARMA (3, 3)-GARCH models. 6th sunspot cycles express the suitable models are ARMA (6, 4), AR (2)-GARCH and ARMA (3, 3)-GARCH process. 7th sunspot cycles depict best fitted models are ARMA (5, 3), AR (3)-GARCH and ARMA (3, 3)-GARCH process. 8th sunspot cycle shows best appropriate models are ARMA (4, 6), AR (2)-GARCH and ARMA (3, 2)-GARCH process. 9th sunspot cycle has best fitted models are ARMA (6, 4), AR (2)-GARCH and ARMA (4, 2)-GARCH process. 10th sunspot cycles express best fitted models are ARMA (6, 4), AR (2)-GARCH and ARMA (4, 4)-GARCH process. 11th sunspot cycle reveals best fitted models are ARMA (6, 4), AR (2)-GARCH and ARMA (5, 1)-GARCH process. 12th sunspot cycle has suitable models are ARMA (5, 3), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 13th sunspot cycle shows best fitted models are ARMA (5, 4), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 14th sunspot cycles express appropriate models are ARMA (4, 6), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 15th sunspot cycles describe the best fitted models are ARMA (5, 3), AR (3)-GARCH and ARMA (3, 3)-GARCH process. 16th sunspot cycles represent suitable models are ARMA (6, 5), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 17th sunspot cycle has best fitted models are ARMA (6, 2), AR (3)-GARCH and ARMA (2, 2)-GARCH process. 18th sunspot cycles describe the appropriate models are ARMA (6, 4), AR (2)-GARCH and ARMA (5, 3)-GARCH process. 19th sunspot cycle has best fitted models are ARMA (4, 3), AR (2)-GARCH and ARMA (5, 3)-GARCH process. 20th sunspot cycles reveal best fitted models are ARMA (4, 3),

AR (2)-GARCH and ARMA (2, 2)-GARCH process. 21st sunspot cycles express suitable models are ARMA (5, 3), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 22nd sunspot cycle has best fitted models are ARMA (4, 5), AR (2)-GARCH and ARMA (6, 1)-GARCH process. 23rd sunspot cycles describe appropriate models are ARMA (3, 4), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 24th sunspot cycles represent suitable models are ARMA (6, 4), AR (2)-GARCH and ARMA (2, 2)-GARCH process. The forecasting evolution of Mean Absolute Error (MAE) shows that ARMA (p, q) model are best fitted and appropriate model as compared to AR(p)-GARCH and ARMA (p, q)- GARCH models. Sunspot cycles have the smallest values of MAE in ARMA (p, q) process. Figure 2 depicts the compression of MAE values of sunspot cycles by using ARMA, AR-GARCH and ARMA-GARCH models.

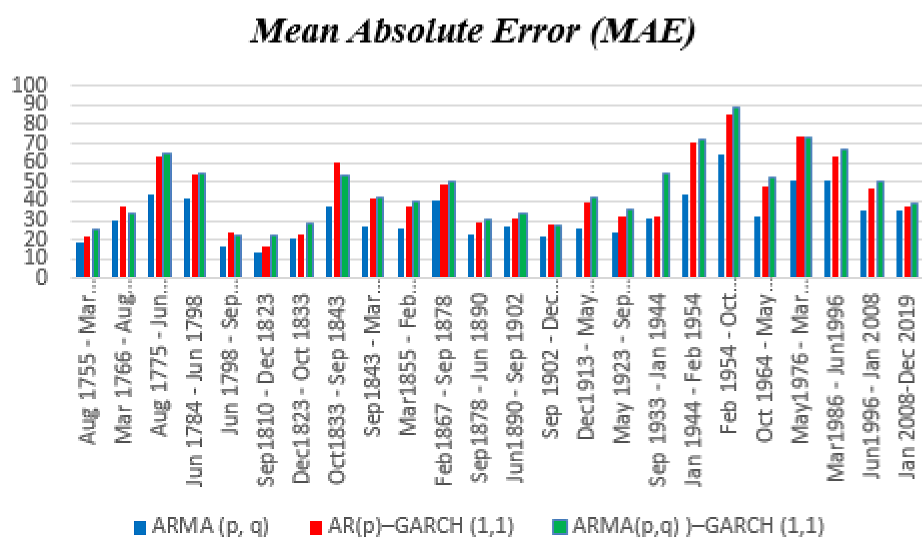

Figure 2: MAE of ARMA, AR-GARCH and ARMA-GARCH process of sunspot cycles.

Mean Absolute Percentage Error (MAPE) has variation in selection of best fitted model among ARMA, AR-GARCH AND ARMA-GARCH process.

Duration	ARMA (p, q)	AR(p)–GARCH (1,1)	ARMA(p,q))–GARCH (1,1)
Aug 1755 - Mar 1766	(4,4) 55.73550	AR(2) 56.7095	(6,5) 56.50119
Mar 1766 - Aug 1775	(2,2) 145.5506	AR(2) 115.7679	(5,1) 124.7012
Aug 1775 - Jun 1784	(5,5) 102.4356	AR(2) 80.41010	(1,1) 88.91661
Jun 1784 - Jun 1798	(3,3) 141.1219	AR(2) 84.66272	(2,2) 82.29495
Jun 1798 - Sep 1810	(5,3) 222.5072	AR(2) 87.63108	(3,3) 85.55745
Sep1810 - Dec 1823	(6,6) 215.0192	AR(2) 81.56241	(3,3) 79.46162
Dec1823 - Oct 1833	(5,5) 125.1342	AR(3) 148.6877	(3,3) 113.0161
Oct1833 - Sep 1843	(3,3) 101.5025	AR(2) 82.81522	(3,2) 79.03446
Sep1843 - Mar 1855	(3,3) 55.33579	AR(2) 59.41300	(4,2) 61.37514
Mar1855 - Feb 1867	(4,4) 149.2235	AR(2) 163.9772	(4,4) 150.4605
Feb1867 - Sep 1878	(3,3) 619.2955	AR(2) 184.8052	(5,1) 163.9832
Sep1878 - Jun 1890	(4,4) 317.3174	AR(2) 110.6451	(2,2) 105.1265
Jun1890 - Sep 1902	(4,4) 351.6125	AR(2) 168.5865	(2,2) 108.7426

Sep 1902 - Dec 1913	(5,5)	469.8335	AR(2)	163.2702	(2,2)	117.5639
Dec1913 - May 1923	(5,5)	124.6412	AR(3)	80.53190	(3,3)	79.70615
May 1923 - Sep 1933	(4,4)	225.7652	AR(2)	136.7743	(2,2)	116.2730
Sep 1933 - Jan 1944	(4,4)	79.25677	AR(3)	94.93236	(2,2)	96.16188
Jan 1944 - Feb 1954	(4,4)	374.3422	AR(2)	127.5823	(5,3)	122.4058
Feb 1954 - Oct 1964	(5,4)	125.9374	AR(2)	101.2019	(5,3)	87.83657
Oct 1964 - May 1976	(3,3)	78.84340	AR(2)	71.04261	(2,2)	74.98162
May1976 - Mar 1986	(3,3)	99.64765	AR(2)	81.00142	(2,2)	80.66910
Mar1986 - Jun1996	(6,5)	120.4492	AR(2)	90.53969	(6,1)	75.91525
Jun1996 - Jan 2008	(5,4)	178.4829	AR(2)	105.7920	(2,2)	93.96324
Jan 2008-Dec 2019	(5,5)	464.3263	AR(2)	238.9409	(2,2)	205.0083

Table 3: Mean Absolute Percentage Error (MAPE) of ARMA, AR-GARCH and ARMA-GARCH of sunspot cycles

Table 3: depict that 1st sunspot cycles has best fitted models are ARMA (4, 4), AR (2)-GARCH and ARMA (6, 5)-GARCH process. Sunspot cycle 1st shows that the best fitted model is an ARMA model with value 55.73550 among these processes. 2nd sunspot cycle has expressed that the appropriate models are ARMA (2, 2), AR (2)-GARCH and ARMA (5, 1)-GARCH process. 3rd sunspot cycle which has suitable fitted models are ARMA (5, 5), AR (2) -GARCH and ARMA (1, 1) -GARCH process. Sunspot cycles 2nd and 3rd represent that the appropriate model is AR-GARCH with value 115.7679 and 80.41010 respectively. 4th sunspot cycle has best fitted models are ARMA (3, 3), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 5th sunspot cycle shows appropriate models are ARMA (5, 3), AR (2)-GARCH and ARMA (3, 3)-GARCH models. 6th sunspot cycles express the suitable models are ARMA (6, 6), AR (2)-GARCH and ARMA (3, 3)-GARCH process. 7th sunspot cycles depict best fitted models are ARMA (5, 5), AR (3)-GARCH and ARMA (3, 3)-GARCH process. 8th sunspot cycle shows best appropriate models are ARMA (3, 3), AR (2)-GARCH and ARMA (3, 2)-GARCH process. Sunspots cycles 4th, 5th, 6th, 7th and 8th reveal that the best model is ARMA-GARCH with value 88.91661, 82.29495, 79.46162, 113.0161 and 79.03446 respectively. 9th sunspot cycle has best fitted models are ARMA (3, 3), AR (2)-GARCH and ARMA (4, 2)-GARCH process. 10th sunspot cycles express best fitted models are ARMA (4, 4), AR (2)-GARCH and ARMA (4, 4)-GARCH process. Sunspot cycles 9th and 10th express that best fitted model is ARMA model with value 55.33579 and 149.2235 respectively among all of these processes. 11th sunspot cycle reveals best fitted models are ARMA (3, 3), AR (2)-GARCH and ARMA (5, 1)-GARCH process. 12th sunspot cycle has suitable models are ARMA (4, 4), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 13th sunspot cycle shows best fitted models are ARMA (4, 4), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 14th sunspot cycles express appropriate models are ARMA (5, 5), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 15th sunspot cycles describe the best fitted models are ARMA (5, 5), AR (3)-GARCH and ARMA (3, 3)-GARCH process. 16th sunspot cycles represent suitable models are ARMA (4, 4), AR (2)-GARCH and ARMA (2, 2)-GARCH process. Sunspot cycles 11th, 12th, 13th, 14th, 15th and 16th analyze that the best model is ARMA-GARCH with values 163.9832, 105.1265, 108.7426, 117.5639, 79.70615 and 116.2730 respectively among these processes. 17th sunspot cycle has best fitted models are ARMA (4, 4), AR (3)-GARCH and ARMA (2, 2)-GARCH process. Sunspot

cycle 17th shows that the best fitted model is ARMA model with value 79.25677 among these models. 18th sunspot cycles describe the appropriate models are ARMA (4, 4), AR (2)-GARCH and ARMA (5, 3)-GARCH process. 19th sunspot cycle has best fitted models are ARMA (4, 3), AR (2)-GARCH and ARMA (5, 4)-GARCH process. Sunspot cycle 18th and 19th express that best fitted model is ARMA-GARCH model with values 122.4058 and 87.83657 respectively. 20th sunspot cycles reveal best fitted models are ARMA (3, 3), AR (2)-GARCH and ARMA (2, 2)-GARCH process. Sunspot cycles 20th and 3rd represent that the appropriate model is AR-GARCH with value 71.04261. 21st sunspot cycles express suitable models are ARMA (3, 3), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 22nd sunspot cycle has best fitted models are ARMA (6, 5), AR (2)-GARCH and ARMA (6, 1)-GARCH process. 23rd sunspot cycles describe in appropriate models are ARMA (5, 4), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 24th sunspot cycles represent suitable models are ARMA (5, 5), AR (2)-GARCH (1, 1) and ARMA (2, 2)-GARCH (1, 1) process. Sunspot cycles 21st, 22nd, 23rd and 24th reveal that the best model is ARMA-GARCH with value 80.66910, 75.91525, 93.96324 and 205.0083 respectively. Most of the cycles follow ARAM-GARCH models are appropriate to model except cycles 2nd, 3rd and 20th shows AR-GARCH models and cycles 1st, 9th, 10th and 17th express is best fitted model are ARMA models. Figure 3 displayed the compression of MAPE values of sunspot cycles by using ARMA, AR-GARCH and ARMA-GARCH models.

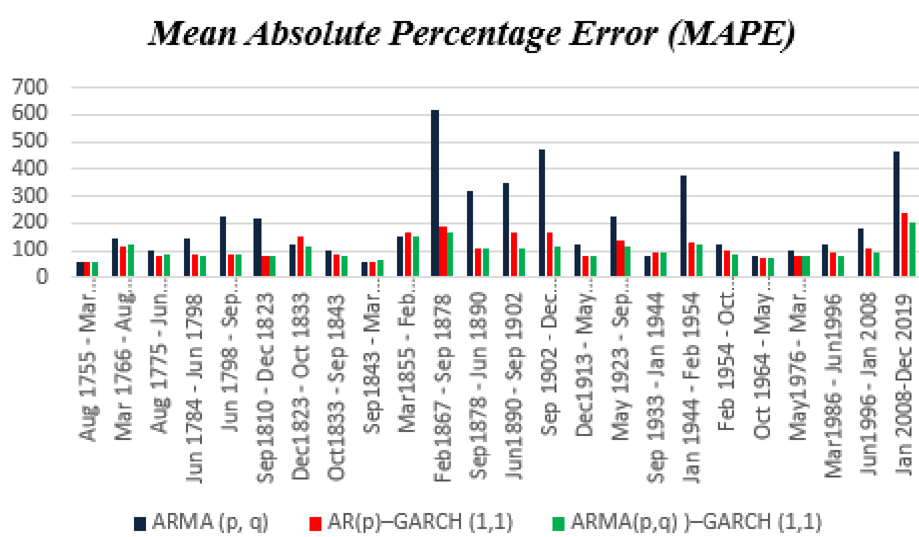

Figure 3: MAPE of ARMA, AR-GARCH and ARMA-GARCH process of sunspot cycles.

Theil's U-statistics test is used to analyze the correlation of observations. Strong correlation shows when U test value near to zero.

Duration	ARMA (p, q)	AR(p)–GARCH (1,1)	ARMA(p,q))–GARCH (1,1)
Aug 1755 - Mar 1766	(6,5) 0.251178	AR(2) 0.34824	(6,5) 0.453746

Mar 1766 - Aug 1775	(5,3)	0.309054	AR(2)	0.501998	(5,1)	0.406759
Aug 1775 - Jun 1784	(6,4)	0.411658	AR(2)	0.859162	(1,1)	0.907096
Jun 1784 - Jun 1798	(5,6)	0.465585	AR(2)	0.837476	(2,2)	0.841223
Jun 1798 - Sep 1810	(6,4)	0.416733	AR(2)	0.999864	(3,3)	0.922685
Sep1810 - Dec 1823	(3,4)	0.418201	AR(2)	0.867214	(3,3)	0.972678
Dec1823 - Oct 1833	(5,3)	0.294667	AR(3)	0.342102	(3,3)	0.486173
Oct1833 - Sep 1843	(4,6)	0.353636	AR(2)	0.863572	(3,2)	0.727303
Sep1843 - Mar 1855	(4,3)	0.296860	AR(2)	0.590749	(4,2)	0.609577
Mar1855 - Feb 1867	(6,4)	0.318617	AR(2)	0.677314	(4,4)	0.766811
Feb1867 - Sep 1878	(6,4)	0.467277	AR(2)	0.845125	(5,1)	0.874876
Sep1878 - Jun 1890	(5,3)	0.384873	AR(2)	0.841212	(2,2)	0.876150
Jun1890 - Sep 1902	(4,3)	0.429832	AR(2)	0.703145	(2,2)	0.836094
Sep 1902 - Dec 1913	(4,6)	0.363862	AR(2)	0.755452	(2,2)	0.744690
Dec1913 - May 1923	(5,3)	0.329163	AR(3)	0.747014	(3,3)	0.826760
May 1923 - Sep 1933	(6,5)	0.340449	AR(2)	0.701884	(2,2)	0.814867
Sep 1933 - Jan 1944	(5,3)	0.336577	AR(3)	0.874060	(2,2)	0.888186
Jan 1944 - Feb 1954	(5,3)	0.366021	AR(2)	0.627014	(5,3)	0.895654
Feb 1954 - Oct 1964	(5,3)	0.483381	AR(2)	0.910967	(5,3)	0.939852
Oct 1964 - May 1976	(4,3)	0.329886	AR(2)	0.921958	(2,2)	0.333498
May1976 - Mar 1986	(5,3)	0.375419	AR(2)	0.908955	(2,2)	0.800143
Mar1986 - Jun1996	(4,5)	0.433141	AR(2)	0.901397	(6,1)	0.772314
Jun1996 - Jan 2008	(3,4)	0.373450	AR(2)	0.925559	(2,2)	0.802811
Jan 2008-Dec 2019	(6,4)	0.370626	AR(2)	0.612406	(2,2)	0.778433

Table 4: Theil's U-Statistics (U test) of ARMA, AR-GARCH and ARMA-GARCH of sunspot cycles

Table 4: depict that 1st sunspot cycles has best fitted models are ARMA (6, 5), AR (2)-GARCH and ARMA (6, 5)-GARCH process. 2nd sunspot cycle has expressed that the appropriate models are ARMA (5, 3), AR (2)-GARCH and ARMA (5, 1)-GARCH process. 3rd sunspot cycle has suitable fitted models are ARMA (6, 4), AR (2) -GARCH and ARMA (1, 1) -GARCH process. 4th sunspot cycle has best fitted models are ARMA (5, 6), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 5th sunspot cycle shows appropriate models are ARMA (6, 4), AR (2)-GARCH and ARMA (3, 3)-GARCH models. 6th sunspot cycles express the suitable models are ARMA (3, 4), AR (2)-GARCH and ARMA (3, 3)-GARCH process. 7th sunspot cycles depict best fitted models are ARMA (5, 3), AR (3)-GARCH and ARMA (3, 3)-GARCH process. 8th sunspot cycle shows best appropriate models are ARMA (4, 6), AR (2)-GARCH and ARMA (3, 2)-GARCH process. 9th sunspot cycle has best fitted models are ARMA (4, 3), AR (2)-GARCH and ARMA (4, 2)-GARCH process. 10th sunspot cycles express best fitted models are ARMA (6, 4), AR (2)-GARCH and ARMA (4, 4)-GARCH process. 11th sunspot cycle reveals best fitted models are ARMA (6, 4), AR (2)-GARCH and ARMA (5, 1)-GARCH process. 12th sunspot cycle has suitable models are ARMA (5, 3), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 13th sunspot cycle shows best fitted models are ARMA (4, 3), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 14th sunspot cycles express appropriate models are ARMA (4, 6), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 15th sunspot cycles describe the best fitted models are ARMA (5, 3), AR (3)-GARCH and ARMA (3, 3)-GARCH process. 16th sunspot cycles represent suitable models are ARMA (6, 5), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 17th sunspot cycle has best

fitted models are ARMA (5, 3), AR (3)-GARCH and ARMA (2, 2)-GARCH process. 18th sunspot cycles describe the appropriate models are ARMA (5, 3), AR (2)-GARCH and ARMA (5, 3)-GARCH process. 19th sunspot cycle has best fitted models are ARMA (5, 3), AR (2)-GARCH and ARMA (5, 3)-GARCH process. 20th sunspot cycles reveal best fitted models are ARMA (4, 3), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 21st sunspot cycles express suitable models are ARMA (5, 3), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 22nd sunspot cycle has best fitted models are ARMA (4, 5), AR (2)-GARCH and ARMA (6, 1)-GARCH process. 23rd sunspot cycles describe appropriate models are ARMA (3, 4), AR (2)-GARCH and ARMA (2, 2)-GARCH process. 24th sunspot cycles represent suitable models are ARMA (6, 4), AR (2)-GARCH and ARMA (2, 2)-GARCH process. The forecasting evolution of Theil's U-statistics test shows that ARMA (p, q) models are best fitted and appropriate model as compared to AR(p)-GARCH and ARMA (p, q)- GARCH models. Sunspot cycles have the smallest value of Theil's U-statistics test in ARMA (p, q) process. Figure 4 displayed that the compression of Theil's U-statistics test values of sunspot cycles by using ARMA, AR-GARCH and ARMA-GARCH models.

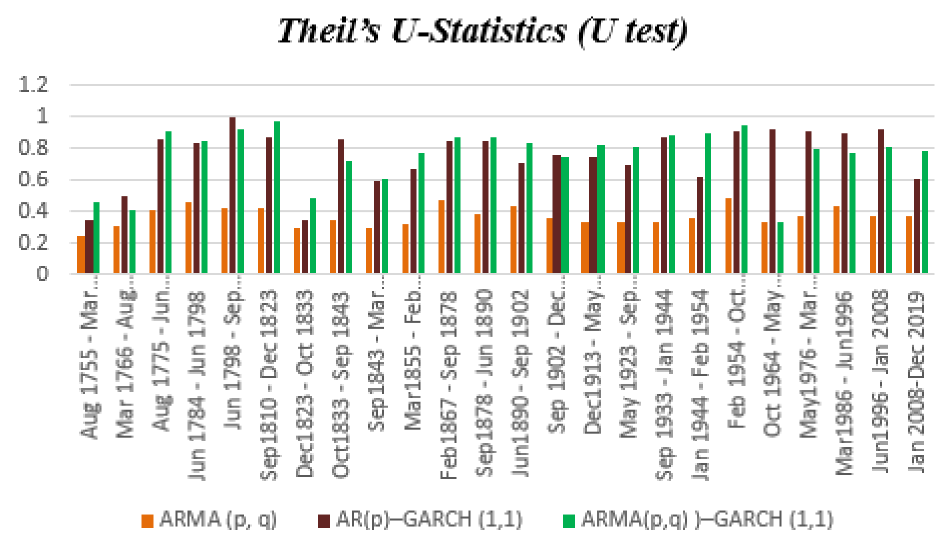

Figure 4: Theil's U-statistics test of ARMA, AR-GARCH and ARMA-GARCH process of sunspot cycles.

4: Conclusion

The novelty of this study is to analyze the forecasting evolution for sunspot cycles using ARMA, AR-GARCH, ARMA-GARCH models. It is already found that the sunspot cycles have stationary nature with second difference and Autocorrelation (AC), Partial Autocorrelation (PAC) and Ljung-Box Q-statistics test are used to check the presence of white noise in the time series data. This study utilizes the stochastic autoregressive and moving average (ARMA) models to forecast the evolution of sunspot cycles. Least Square Estimation is used to investigate the ARMA process. The stationary generalized autoregressive conditional heteroskedasticity GARCH (1, 1) volatility model with specification autoregressive AR (p) process and ARMA (p, q) is used to forecast the evolution of sunspot cycles. The selection of appropriate models is based on the smallest value of the Durbin-Watson statistics test. Durbin-Watson (DW) statistics test value of each sunspot cycle is less than 2 which shows that sunspot observations are correlated to each other. Under model Identification and Estimation, diagnostic checking and Forecasting the ARMA (p, q), AR (p)-GARCH and ARAM (p, q)-GARCH models are found to be the most appropriate. The Gaussian quasi maximum likelihood estimation (QMLE) is used to estimate GARCH (1, 1) process for the specification of AR (p) and ARMA (p, q) models. The selection of the appropriate model is based on residual diagnostic checking's such as ARCH LM, normality test and correlogram squared residuals. Forecasting evolutions are verified by Root Mean Squared Error (RMSE), Mean Absolute Error (MAE), Mean Absolute Percentage Error (MAPE) and Theil's U-Statistics test (U test). The forecasts for the evolution of sunspot cycles obtained by ARMA, AR-GARCH and ARMA-GARCH models are compared. RMSE, MAE and U test is utilized to check the appropriateness of various ARMA models. Only the MAPE exhibited the appropriateness of ARMA-GARCH model apart from cycles 1^{st}, 9^{th}, 10^{th} and 17^{th} follows ARMA models and cycles 2^{nd}, 3^{rd}, 4^{th}, 5^{th}, 6^{th} and 20^{th} follows AR-GARCH model.

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