Quantum Interference of Eigenfunctions for Spin Dynamics in Topological Space-Time

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Abstract

Eigenfunctions for quantum superposition of precessing spin Dirac Bosons at 0.4 ≤ n_f≤ 0.9, Dirac spinning Fermions at 0.1 ≤ n_f≤ 0.3 and Dirac Fermions with modified Hamiltonian operator (Hermitian), H_{op} (Topological Space-Time) = $\left\{-\frac{\hbar^2}{2m}\nabla^2_{topo} + \frac{\partial}{\partial t} < \left|V_{(r_{op},t)}\right| >\right\}$, are obtained. The gyroscopic constant, $0.02 \leq g^2/\hbar c \leq 0.08$ is a manifestation of whirlpools of various sizes of energy field fluids because electrons are not point particles especially in topological space-time with braided configurations.

For an unperturbed electronic system, there is an optimal variation of orbit-spin coupling, $\frac{e^2}{\hbar c} = \frac{1}{137}$, Bohr's magneton ($\mu_B = \frac{e\hbar}{2m_e c}$ and of spectroscopic splitting factor, (g_e = 2.0023) due to 'fractional charge quantization' corresponding to 'fractional masses distribution' in braided energy field fluid configurations. QED (quantum electrodynamic) behavior of eigenfunctions at 0.1 ≤ n_f≤ 0.9 is a manifestation of dipole helicon spin surfaces due to electron-hole pairs.

Eigenfunctions, in totality, is defined as

$$\psi_{spindyn}\left(r_{op}, \alpha, t_{n_f}\right) = \mu_B.\frac{g^2}{\hbar c}\left(\frac{0.17 \leq \alpha \leq 1.53\, rad}{\sqrt{\pi}2^{nf}}\right)^{1/2}$$

where $\mu_B$is the Bohr's magneton, $H_{n_f} = 2^{n_f}$ the Hermite polynomials (Hermitian) for the integrated oscillators (twigs) in braided configurations at 0.1 ≤ n_f≤ 0.9.

Results and Discussions

The fractional Fourier transform (FRFT) of order α of x(t) is deciphered by Almeida [1]

$$\left.\begin{array}{l} F_\alpha[x(t)]= X_\alpha(u)= \int_{-\alpha}^{\alpha} x(t)K_\alpha(t,u)dt= \sqrt{\frac{1-jcot\alpha}{2\pi}}\, e^{j\frac{u^2}{2}cot\alpha} \int_{-\alpha}^{\alpha} x(t)e^{j\frac{u^2}{2}cot\alpha-jutcoseca} \\ \text{,if } \alpha \text{ is not a multiple of } \pi, x(t), \text{if } \alpha \text{ is a multiple of } 2\pi \text{ and } x(-t), \text{if } \alpha+\pi \text{ is a multiple of } 2\pi \end{array}\right\} \quad (1)$$

where α is a rotation angle in time-frequency plane and F_α is the FRFT operator. For α = π/2 the kernel, K_α coincides with the kernel of Fourier transform (FT). Using equation (1), we obtained Hermite function for the integrated oscillators in a braided configuration at fractional quantum numbers, $0.1 \leq n_f \leq 0.9$ [2].

$$H_{n_f}(F_\alpha(x(t)) = 2^{n_f}, \quad 0.1 \leq n_f \leq 0.9 \quad (2)$$

Equation (2) is consistent with Hermite polynomials. The eigenfunction for twisted and twigged electron quanta [2] is

$$\psi_{n_f} = e^{in_f\alpha} = \left(\frac{0.17 \leq \alpha \leq 1.53\ rad}{\sqrt{\pi}2^{n_f}}\right)^{1/2} \quad (3)$$

The twisted and twigged electron quanta is a result of stretching with 'quantization of events' for fractional masses (twigs or knots) in braided energy field configurations. We obtained the expression for stretching [3]

$$\psi_{stretching}\ (\alpha \equiv \emptyset) = \frac{i\hbar t_{n_f}}{m_{n_f}} \quad (4)$$

where α(ω, t) ≡ azimuth angle, ∅; $\hbar = \frac{h}{2\pi}$ is Planck's constant, m_{n_f} the fractional masses (twiggs or oscillators) in a braided configuration and t_{n_f} the fractional times for corresponding frequencies.

Combining equations (3) and (4), we have

$$\psi_{n_f}(r_{op},\alpha,t) = \frac{i\hbar t_{n_f}}{m_{n_f}}\left(\frac{0.17 \le \alpha \le 1.53\, rad}{\sqrt{\pi}2^{n_f}}\right)^{1/2} \quad (5)$$

$\hbar t_{n_f}$ in equation (5) defines 'quantization of events'. Defining

$$e_{n_f} = \sum_{n_f=0.1}^{0.9} n_f \,.\, e \text{ and } m_{n_f} = \sum_{n_f=0.1}^{0.9} n_f\, m_e \quad (6)$$

where e is the charge of an electron, $m_e$is the mass of an electron, $e_{n_f}$the fractional charge of an electron and $m_{n_f}$the fractional mass of an electron.

With n_f x $g^2/\hbar c$, whirlpool of precessional spin waves are produced. For such trivial situations, we obtained eigenfunctions of whirlpools of diverse categories of sizes of energy field fluids equivalent to potential wells with strips in its depths labeled as $0.1 \le n_f \le 0.9$[2].

$$\left.\begin{array}{c} \psi_{whirlpools}\left(r_{op},l,t\right)=\hbar l r_e^{-l}.\left(\frac{\pi}{2Kr}\right)^{\frac{1}{2}} J_{l+\frac{1}{2}}(K_r), r< r_e \\ \text{with } K=\left[\frac{2\mu}{\hbar}\left(E+\frac{\partial}{\partial t}|V(r,t)|\right)\right]^{\frac{1}{2}}, \mathrm{V(r,t)}=\frac{{e_{n_f}}^2}{r_e} \text{ and} \\ K=\left[k_{n_f}^2 - \frac{4\pi\mu n_f e_{n_f}}{\hbar r_e}\right] \end{array}\right\} \quad (7)$$

where r_e is the radius of point electron, l the azimuth quantum number, μ the reduced or fractional masses, $J_{l+\frac{1}{2}}(K_r)$ the modified Bessel function and $\frac{\partial}{\partial t}|V(r,t)|$ the varying depths of the whirlpools. $\frac{g^2}{\hbar c}$ is the equivalent of the eigenfunctions defined in equation (7). The vector matrix n_f x $g^2/\hbar c$ will yield whirlpools of varying sizes of energy field fluids of electron in topological space-time. This confirms to the fact that the fundamental constants change into variables especially in topological space-time. The magnetic moment of an electron or of an atom or ion in a free space is:

$$\mu_m = \gamma\hbar\bar{J} = -g_e\mu_B\bar{J},\ \hbar\bar{J} = \hbar\bar{L} \pm i\hbar\bar{S} \quad (8)$$

where the total angular momentum, $\hbar\bar{J}$ is the sum of orbital, $\hbar\bar{L}$ and spin, $\hbar\bar{S}$ angular momenta. The constant γ is the ratio of the magnetic moment to the angular momentum, will change with 'fractional charge quantization' at $0.1 \le n_f \le 0.9$ in topological space-time. For electronic systems, g_e the spectroscopic splitting factor is defined as

$$g_e \mu_B \equiv -\gamma\hbar \quad (9)$$

With changes in γ, $g_e\mu_B$ will also change. The spectroscopic splitting factor, $g_e(= 2.0023)$ is different from the gyroscopic coupling constant, $0.02 \le \frac{g^2}{\hbar c} \le 0.08$. The electron coupling constant, $\frac{e^2}{\hbar c} \sim \frac{1}{137}$ is specifically meant for orbit-spin coupling. When spinning changes into precessional spin quantum states ($\Phi = e^{\pm im\emptyset}$; m is the magnetic quantum number, ϕ the azimuth angle, $\Phi = e^{\pm im\alpha}$, $\alpha \equiv \emptyset$, $0.17 \le \alpha \le 1.53\, rad$), the gyroscopic coupling constant becomes overwhelming as compared to orbit spin coupling constant, $\frac{e^2}{\hbar c}$. The Landau factor, g_e is defined as

$$\left.\begin{array}{c} g_e = 1 + \frac{\bar{J}(\bar{J}+1)+S(\bar{S}+1)-\bar{L}(\bar{L}+1)}{2\,\bar{J}(\bar{J}+1)}; \quad \hbar\bar{J} = \hbar\bar{L} \mp i\hbar\bar{S} \\ \Delta n = \mp 1, \mp 2, \mp 3, \ldots\ldots selection\ rule\ for\ whirlpools \end{array}\right\} \quad (10)$$

With our analysis, we can say that $\bar{L}\,\bar{S}\ or\ \bar{J}$ coupling in an unperturbed electronic system changes optimally with fractional charge quantization and so is the case for g_e (as in equation (10)). For

$$g_e = 0, \hbar\bar{L} = 0 \implies \hbar\bar{J} = \mp i\hbar\bar{S}, \text{ [as in equation (10)]} \quad (11)$$

$\bar{S}$, .i.e., spin matrix or a vector spin matrix in equation no.(11) corresponds to Panli-spin matrices and other matrices for allowed spin-energy states. With $\hbar\bar{J} = \mp i\hbar\bar{S}$, the lowest spin energy state is :

$$\left.\begin{array}{c} m_j = \mp\frac{1}{2} \\ we\ ascribe \quad \psi_{spin\ dynamics}\left(r_{op}, \alpha, t_{n_f}\right) = \psi_{space} \cdot \psi_{charge} \cdot \psi_{spin} \\ where \quad \psi_{space} = |\psi\left(r_{op}, \alpha, t_{n_f}\right)> e^{-i\hbar.\omega.t_{n_f}} \\ and\ \psi_{charge} \cdot \psi_{spin} \underset{\alpha\equiv\emptyset}{\sim} e^{\pm im\emptyset}, \end{array}\right\} \quad (12)$$

where $\psi_{space} = |\psi\left(r_{op}, \alpha, t_{n_f}\right) > e^{-i\hbar.\omega.t_{n_f}}$ is the ket-state function for topological space-time with diverse categories of topological phases, $e^{-i\hbar.\omega.t_{n_f}}$ with 'quantization of events', $\hbar.t_{n_f}$ in equation (12). Topological phases are related with time crystals like the time-crystals of electrons or holes preferably with spin-dynamics.

Considering the general representation of

$$\left.\begin{array}{c}\psi_{space}(r_{op},\alpha,t)=C\ e^{\mp i\hbar.\omega.t_{n_f}}\\ \psi_{space}(r_{op},\alpha,t)=C\ |e^{\mp i\hbar.\omega.t_{n_f}}>\\ \psi^{*}_{space}(r_{op},\alpha,t)=C^{*}|\ e^{\mp i\hbar.\omega.t_{n_f}}>\end{array}\right\} \quad (13)$$

where C, C* are complex constants and operators in topological space-time, which if normalized can yield energy eigenvalues of spin dynamics. We also know

$$\left.\begin{array}{c}E_n=(n+1/2)h\upsilon,\ \ n=0,1,2,3,......\\ For\quad E_o=\mp\frac{1}{2}h\upsilon\ with\ m_j=\mp 1/2\\ E_{n_f}=n_f\ h\nu=\left(0.1\le n_f\le 0.9\right)h\nu\end{array}\right\} \quad (14)$$

Using the reference book of quantum mechanics [4, 5], we consider

$$\left.\begin{array}{c}\psi(r_{op},\alpha,t)=\exp\left[\frac{-i}{\hbar}(p_{op}.r_{op}\mp E\ \mathrm{x}\ \mathrm{t}\right]\equiv\exp[-i\hbar(\overline{p}.\overline{r}\mp\omega t_{n_f}]\\ \equiv\left(\frac{0.17\le\alpha\le 1.53\ rad}{\sqrt{\pi}2^{n_f}}\right)^{1/2}\\ where\ p_{op}=-i\,\hbar\nabla,\ \ r_{op}\equiv\hbar=\delta_{Dirac}(r-r_o),\ \ E_{op}=i\hbar\frac{\partial}{\partial t}\\ \omega=2\pi\nu,\ \ E=h\upsilon\end{array}\right\} \quad (15)$$

where $e^{-i\hbar.\omega.t_{n_f}}$ in equation (15) deals with topological phases of time crystals of electrons or holes.

Let

$$\left.\begin{array}{c} |\,\psi\left(r_{op},\alpha,t_{n_f}\right) > e^{-i\hbar.\omega.t_{n_f}} \\ <\psi^*\left(r_{op},\alpha,t_{n_f}\right)|\; e^{-i\hbar.\omega.t_{n_f}} \\ H_{n_f}(op)|\psi\left(r_{op},\alpha,t_{n_f}\right) > e^{-i\hbar.\omega.t_{n_f}}, \\ e^{-i\hbar.\omega.t_{n_f}} <\psi^*\left(r_{op},\alpha,t_{n_f}\right)|\,H^*{}_{n_f}(op) \end{array}\right\} \quad (16)$$

The mathematical expressions of equations (16) for the braided configuration of energy fields of precessing spin electrons with $H_{n_f}(op) = 2^{n_f}$ (fractional Hermite polynomials) for 'oscillators' at knots, $0.1 \le n_f \le 0.9$ with be specifically meant for time crystals in topological space-time. H_{n_f} becomes an operator and is Hermitian.

$$I = < H_{n_f}(op)|H^*{}_{n_f} >, 0.1 \le n_f \le 0.9 \quad (17)$$

where I is the involution or identity operator.

The same principle works for $g_e = 0$, but with $m_j = \mp 1/2$ for pure spinning electrons ($\hbar\bar{J} = \mp i\hbar\bar{S}, with\ \hbar\bar{L} = 0$) without time crystals of electrons (without topological space-time and topological phases). The modified Hamiltonian for $\mp i\hbar\bar{S}$, a vector space for spin matrices including Pauli-spin matrices will be used. Let the modified Hamiltonian which is Hermitian operator too, be used

$$H_n(op) = \left\{\frac{-\hbar^2}{2m}\nabla^2 + < |V(r,t)| >\right\} = \left\{\begin{matrix} \frac{1}{2} & 0 & 0 & 0 & 0 & 0\ldots \\ 0 & \frac{3}{2} & 0 & 0 & 0 & 0\ldots \\ 0 & 0 & \frac{5}{2} & 0 & 0 & 0\ldots \\ 0 & 0 & 0 & \frac{7}{2} & 0 & 0\ldots \\ 0 & 0 & 0 & 0 & \frac{9}{2} & 0\ldots \\ 0 & 0 & 0 & 0 & 0 & \frac{11}{2}\ldots \end{matrix}\right\} \quad (18)$$

Thus $$< H_n(op)\,|\,H_n{}^*(op) > = I \quad (19)$$

We will consider the superposition of time crystals of electrons or holes in a braided configuration of topological space-time accompanied with topological phases leading to precession or gyroscopic behavior of whirlpools knows as 'quantum chiral states' but different from 'chern' quantum states as in the case of fractional quantum Hall effect. Phase changes of topological space-time defining time-crystals of electrons are different from generalized space-time.

With $\psi_{n_f}(r_{op}, \alpha, t)$ plotted against $0.1 \leq n_f \leq 0.9$ by using equation (3), we get transition at $n_f = 0.4$, therefore, separate $0.4 \leq n_f \leq 0.9$ for Dirac Bosons (quasiparticles or time-crystals of electrons) and $0.1 \leq n_f \leq 0.3$ for Dirac Fermions or Majorana Fermions for electron-hole pairs (quasiparticles or time crystals of electrons). The $\psi_{n_f}(r_{op}, \alpha, t)$ profile shows linear behavior for $0.1 \leq n_f \leq 0.3$ and logarithmic behavior for $0.4 \leq n_f \leq 0.9$ for quasiparticles or time crystals of electrons preferentially obeying the QED (quantum electrodynamic theory). Quasiparticles of electrons with spinning Dirac Bosons and Dirac Fermions or Majorana Fermions of electron-hole pairs obey the following equations of QED theory [6].

$$\left.\begin{array}{ll} \emptyset(\bar{r},t)= 0, & \overline{A}(\bar{r},t) = -\hbar\, n_f\, \alpha(t) \\ \overline{E}\,(\bar{r},t) \approx \sqrt{2}\, n_f \alpha\omega\hbar, & \overline{B}(\bar{r},\ t) \cong -\sqrt{2}\, n_f \hbar\theta sin\theta \overline{\nabla}_r \alpha(t) \\ 0.17 \leq \alpha(t) \leq 1.53\ rad, & 0.1 \leq n_f \leq 0.9 \end{array}\right\} \tag{20}$$

where in equation (2), $\theta sin\theta\overline{\nabla}_r\alpha(t)$ represents the helical pattern of dipole radiations, here in this case, the quasiparticles of electron-hole pairs. The oscillatory term with natural frequency ω_o , .i.e., $-\omega_o^2\left[\overline{A}e^{2i(kr-\omega t)} + \overline{A_o^*}e^{-2i(kr-\omega t)}\right]$ in QED theory [6] is responsible to producing quasiparticles of electrons with Dirac spinning Bosons at knots or twigs and Dirac Fermions or Majorana Fermions with electron-hole pairs in braided configurations for topological space-time. Remember, space-time is warped, an emergent phenomena and defined with diverse categories of geometries.

Let us define,

$$E_{op}|\Psi\left(r\ \uparrow\downarrow, \alpha, t_{n_f}\right)>_{0.1 \leq n_f \leq 0.9} e^{-i\hbar\omega t_{n_f}}$$

$$= i\hbar\frac{\partial}{\partial t}|\Psi\left(r\ \uparrow\downarrow, \alpha, t_{n_f}\right)>_{0.1 \leq n_f \leq 0.9} e^{-i\hbar\omega t_{n_f}} \tag{21}$$

where the term $e^{-i\hbar\omega t_{n_f}}$ corresponds to topological phases of quasiparticles of electrons. For each ket-states function, there is a complex conjugate bra-state function.

$$E_{op} < \Psi^*\left(r\ \uparrow\downarrow, \alpha, t_{n_f}\right)|_{0.1 \leq n_f \leq 0.9}\ \ e^{-i\hbar\omega t_{n_f}}$$

$$= i\hbar\frac{\partial}{\partial t} < \Psi^*\left(r\ \uparrow\downarrow, \alpha, t_{n_f}\right)|_{0.1 \leq n_f \leq 0.9}\ e^{-i\hbar\omega t_{n_f}} \qquad (22)$$

Dirac –δ function follows inversion symmetry. Quantum commutation of bra-ket state functions yield normalization, .i.e., Dirac –δ symmetry operators are to be normalized with fractional Fourier (FTFR) transform operators in (ω, α) planes.

$$E_{op}|\Psi\left(r\uparrow\downarrow, \alpha, t_{n_f}\right) >_{0.1 \leq n_f \leq 0.9} = C(0,t)e^{\mp i\hbar\omega t_{n_f}}$$

$$= i\hbar\frac{\partial}{\partial t}|\Psi\left(r\uparrow\downarrow, \alpha, t_{n_f}\right) >_{0.1 \leq n_f \leq 0.9} \qquad (23)$$

where in equation (23), $r\uparrow\downarrow$ shows momentum operator $p_{op} = -i\hbar\,\overline{\nabla}_r$ with magnetic excitations upwards and downwards for precessing spin electrons, .i.e., gyroscopic behavior.

In such trivial situations, $\hbar\bar{S}$ or $\mp i\hbar\bar{S} = 0$ with $m_j = \mp\frac{1}{2}$.

Thus, quasiparticles are the manifestations of whirlpools due to gyroscopic behavior. These whirlpools are specifically meant for Dirac spinning Bosons and for Dirac Fermions, quite different from, $\mp i\hbar\bar{S}$ (including Pauli-spin matrices). Equations (1), (3) and (7) together define eigenfunctions for stretching, twisting, twigging and gyroscopic behavior. Stretching of energy field fluids associated with electrons is embedded with twisting and twigging phenomena. Replacement of equation (7) is defined for $0.02 \leq \frac{g^2}{\hbar c} \leq 0.08$ and that n_f x $\frac{g^2}{\hbar c}$ will yield a vector matrix for varying sizes of energy field fluid whirlpools. The Bohr's magneton will also change for quasiparticles, .i.e., $\frac{e_{n_f}\hbar}{2m_{n_f}c}$ [following equation (6)]. In other words, the constants like the Bohr's magneton, g_e the spectroscopic splitting factor (2.0023), electron coupling constant, $\frac{e^2}{\hbar c}$ and $0.02 \leq \frac{g^2}{\hbar c} \leq 0.08$ become quantum operators, complex numbers, complex constants and follow the quantum commutation rules. The Bohr's magneton, a constant has a fixed value of 9.274 x 10^{-24} Joules/ Tesla or 58 x 10^{-6}eV/ Tesla. These constants are, however, no more valid for quasiparticles but used with 'fractional charge quantization' for corresponding 'fractional masses' of twigs or time crystals of electron-hole pairs. 'Quantization of events', $i\hbar t_{n_f}$ lead to time crystals in the form of energy field fluids.

Thus $$< \Psi(r\uparrow\downarrow, \alpha, t)|U|\Psi(r\uparrow\downarrow, \alpha, t) >_{0.1 \leq n_f \leq 0.9} \qquad (24)$$

will yield matrix tensor Dirac-δ with symmetry operators

$$< \Psi(r\uparrow\downarrow, \alpha, t)|U^*|\Psi(r\uparrow\downarrow, \alpha, t) >_{0.1 \leq n_f \leq 0.9} \qquad (25)$$

With quantum commutation of unitary operators in Hilbert space, .i.e., normalization with

$$UU^* = I^2 = 1 \qquad (26)$$

where I is the involution operator.

For Dirac spinning Bosons (quasiparticles at knots or twigs) at $0.4 \leq n_f \leq 0.9$ every ket-state functions which is already symmetric can be found by superposition of the bra-state function accompanied with the topological phases like:

$$\left.\begin{array}{c} |\Psi(r\uparrow\downarrow,\alpha,t_{n_f})>_{0.4 \leq n_f \leq 0.9} \text{ x } <\Psi(r\uparrow\downarrow,\alpha,t_{n_f})>_{0.4 \leq n_f \leq 0.9}|e^{-i\hbar\omega t_{n_f}}, \\ \frac{e^{-i\hbar\omega t_{n_f}}}{\sqrt{2}}\left[|\Psi(r\uparrow\downarrow,\alpha,t_{n_f})>_{0.4 \leq n_f \leq 0.9} \text{ x } <\Psi(r\uparrow\downarrow,\alpha,t_{n_f})_{0.4 \leq n_f \leq 0.9}| + <\Psi(r\uparrow\downarrow,\alpha,t_{n_f}) \mid \Psi(r\uparrow\downarrow,\alpha,t_{n_f})>_{0.4 \leq n_f \leq 0.9}|\right], \\ and\ |\Psi^*(r\uparrow\downarrow,\alpha,t_{n_f})>_{0.4 \leq n_f \leq 0.9} \text{ x } <\Psi^*(r\uparrow\downarrow,\alpha,t_{n_f})>_{0.4 \leq n_f \leq 0.9}|e^{-i\hbar\omega t_{n_f}} \end{array}\right\} \qquad (27)$$

For reflection symmetry, the same equation $< \Psi\left(r\uparrow\downarrow, \alpha, t_{n_f}\right) | \Psi\left(r\uparrow\downarrow, \alpha, t_{n_f}\right) >$ becomes

$$\left.\begin{array}{c} |\Psi\left(r\downarrow\uparrow,\alpha,t_{n_f}\right)>_{0.4\le n_f\le 0.9} \text{ x } <\Psi\left(r\downarrow\uparrow,\alpha,t_{n_f}\right)>_{0.4\le n_f\le 0.9}|e^{-i\hbar\omega t_{n_f}}, \\ \frac{e^{-i\hbar\omega t_{n_f}}}{\sqrt{2}}\left[|\Psi\left(r\downarrow\uparrow,\alpha,t_{n_f}\right)>_{0.4\le n_f\le 0.9} \text{ x } <\Psi\left(r\downarrow\uparrow,\alpha,t_{n_f}\right)_{0.4\le n_f\le 0.9}| + <\Psi\left(r\downarrow\uparrow,\alpha,t_{n_f}\right)|\Psi\left(r\downarrow\uparrow,\alpha,t_{n_f}\right)>_{0.4\le n_f\le 0.9}|\right], \\ and\ |\Psi^*\left(r\downarrow\uparrow,\alpha,t_{n_f}\right)>_{0.4\le n_f\le 0.9} \text{ x } <\Psi^*\left(r\downarrow\uparrow,\alpha,t_{n_f}\right)>_{0.4\le n_f\le 0.9}|e^{-i\hbar\omega t_{n_f}} \end{array}\right\} \quad (28)$$

where $|\Psi^*(r\downarrow\uparrow,\alpha,t) > and < \Psi^*(r\downarrow\uparrow,\alpha,t)|$ are implicit ket-and-bra-state functions. For antisymmetric Dirac-δ functions, we ascribe the Dirac Fermions or Majorana Fermions with electron-hole pairs at $0.1 \le n_f \le 0.3$ following the QED theory for dipole quasiparticles of electrons and holes.

$$\frac{e^{-i\hbar\omega t_{n_f}}}{\sqrt{2}}\left[\begin{array}{c}|\Psi\left(r\uparrow\downarrow,\alpha,t_{n_f}\right)>_{0.1\le n_f\le 0.3} \text{ x } <\Psi\left(r\uparrow\downarrow,\alpha,t_{n_f}\right)_{0.1\le n_f\le 0.3}| - \\ <\Psi\left(r\uparrow\downarrow,\alpha,t_{n_f}\right)_{0.4\le n_f\le 0.9}|\Psi\left(r\uparrow\downarrow,\alpha,t_{n_f}\right)>\end{array}\right] \quad (29)$$

For reflection symmetry

$$\frac{e^{-i\hbar\omega t_{n_f}}}{\sqrt{2}}\left[\begin{array}{c}|\Psi\left(r\downarrow\uparrow,\alpha,t_{n_f}\right)>_{0.1\le n_f\le 0.3} \text{ x } <\Psi\left(r\downarrow\uparrow,\alpha,t_{n_f}\right)_{0.1\le n_f\le 0.3}| - \\ <\Psi\left(r\downarrow\uparrow,\alpha,t_{n_f}\right)_{0.4\le n_f\le 0.9}|\Psi\left(r\downarrow\uparrow,\alpha,t_{n_f}\right)>\end{array}\right] \quad (30)$$

Recently, time crystals of electron are experimentally observed and are used on quantum computers [7, 8]. Time crystals or quasiparticles of electrons and holes are made real [9].

We established the shapes of eigenfunctions following the bra-and-ket-state functions, their superposition quantum states, with QED theory for quasiparticles of electron-hole pairs in the form of Dirac precessions spinning Bosons for electron-hole pairs and Dirac Fermions or preferably Majorana Fermions with electron-hole pairs. The modified Hamiltonian yields a vector space of $\mp i\hbar\bar{S}$ (including Pauli spins) for electrons only and quasiparticles with

$\frac{\partial}{\partial t} < |V(r,t)| >$ in the momentum k-space with whirlpools of energy field fluids.

Thus the eigenfunctions for spin dynamics of electrons produce vector space of $\mp i\hbar\bar{S}$ and vector space of 'quasiparticles or time-crystals' of electron-hole pairs following the QED dipole radiations in the k-momentum space for spinning Bosons at 'twigs or knots' and Majorana Fermions without 'knots' in braided configurations of electron-energy field fluids.

Flow dynamics of electron changes its configurations like with stretching, twisting, twigging (knots) and gyroscopic behavior. The vector space preferably the topological space (n_f x $\frac{g^2}{\hbar c}$) yields 'quasiparticles' with $\left(\frac{0.17 \le \alpha \le 1.53\, rad}{\sqrt{\pi}2^{n_f}}\right)^{1/2}$ in the form of energy field fluid whirlpools. μ_B, the Bohr's magneton remains same or constant for $\mp i\hbar\bar{S}$ and $\hbar\bar{J} = \hbar\bar{L} \pm i\hbar\bar{S}$ but changes for quasiparticles for $\mp i\hbar\bar{S} = 0$. Thus, the energy eigenvalues of such electron dynamics can be evaluated with eigenfunctions in braided energy field fluid configurations.

$$\psi_{spindyn}\left(r_{op}, \alpha, t_{n_f}\right) = \mu_B.\frac{g^2}{\hbar c}\left(\frac{0.17 \le \alpha \le 1.53\, rad}{\sqrt{\pi}2^{n_f}}\right)^{1/2} \tag{31}$$

$$\left.\begin{array}{c} where\;\; H_{n_f} = 2^{n_f}, \qquad H_{n_f} < \frac{\partial}{\partial t}\left|V\left(r, t_{n_f}\right)\right| >= 2^{n_f} \\ 0.02 \le \frac{g^2}{\hbar c} \le 0.08, \qquad 0.1 \le n_f \le 0.9 \\ \mu_B,\;\; constant\left(\frac{9.274x10^{-24}\; Joules}{Tesla}\right) or \\ 58\; x\; 10^{-6}\frac{eV}{Tesla} \\ \mu_B\; (quasiparticles) is\; no\; more\; constant. \end{array}\right\} \tag{32}$$

With equation (31), energy eigenvalues can be determined replacing μ_B, the Bohr's magneton with E_g, band energy gaps of materials under consideration.

For spinning electrons in topological space-time, the Hamiltonians operator

$H_{op}(\hbar\bar{L} \pm i\hbar\bar{S} = \hbar\bar{J})$ is Hermition. But $\frac{\partial}{\partial t} < |V(r,t)| >$ becomes zero and changes into $< |V(r,t)| >$ for $\bar{J}\,\bar{S}$-coupling or $\bar{L}\,\bar{S}$-coupling. $H_{n_f}(\pm i\hbar\bar{S} = 0)$ becomes zero and is replaced by $H_{(op)_{\left(n+\frac{1}{2}\right)}}(\hbar\bar{J}) = \left\{-\frac{\hbar^2}{2m}\nabla^2 + |V(r,t)|\right\}$. In such trivial situations, equation (31) is modified:

$$\psi_{spindyn}(\hbar \bar{J}) = \mu_B . \frac{e^2}{\hbar c}\left(\frac{0.17 \leq \alpha \leq 1.53\ rad}{\sqrt{\pi}2^{\left(n+\frac{1}{2}\right)}}\right)^{1/2}; [following\ equation\ (14)] \quad (33)$$

where

$$H_{\left(n+\frac{1}{2}\right)}(\hbar \bar{J} = \hbar \bar{L} \pm i\hbar \bar{S}) = 2^{\left(n+\frac{1}{2}\right)}; \quad n = 0,1,2,3,\ldots\ldots \qquad and \qquad \frac{e^2}{\hbar c} = \frac{1}{137} \quad (34)$$

For equation (33), μ_B the Bohr's magneton will remain constant.

Both equations (31) and (34) are applicable for time-crystals or 'quasiparticles of electron-hole pairs' with certain conditions outlined above.

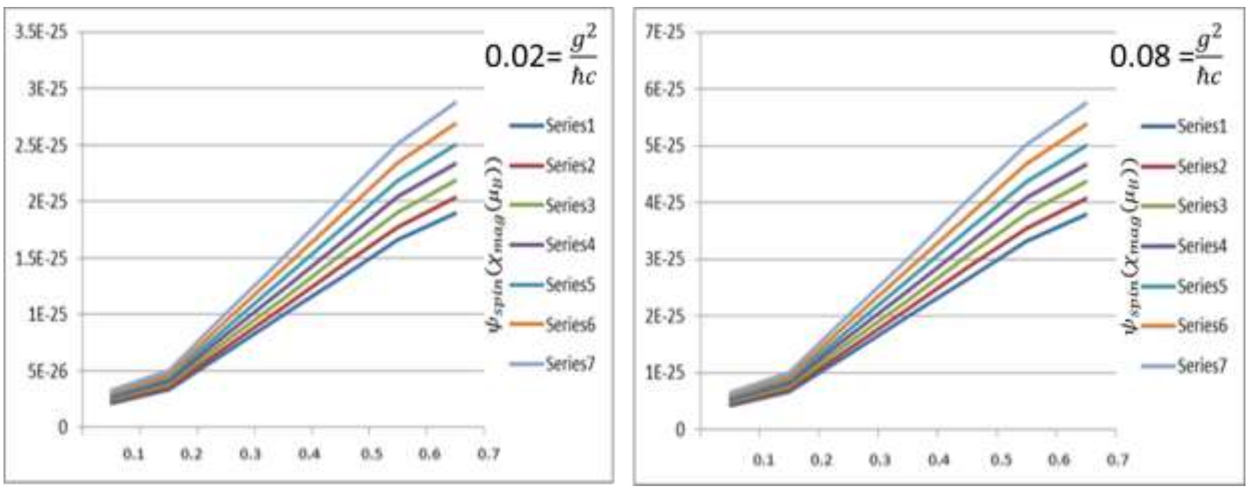

Figure:1. Spin eigenfunctions for $0.17 \leq \alpha \leq 1.53\ rad$ at $0.1 \leq n_f \leq 0.9$ for $\frac{g^2}{\hbar c}$ with 0.02 and 0.08, respectively for μ_B, the Bohr's magneton ($58\ x\ 10^{-6}\ \frac{eV}{Tesla}$).

Figure 1 shows the energy field fluids for whirlpools and swirling potential barriers with $\frac{g^2}{\hbar c}$ at 0.02 and 0.08, respectively for quantum fields at fractional quantum states, $0.1 \leq n_f \leq 0.9$ corresponding to dipole behavior of interacting spins (quantum chirals) with QED [6] preferably due to 'quantum tunneling' and 'quantum fluctuations', (topological phases) in topological space-time (emerging phenomena).

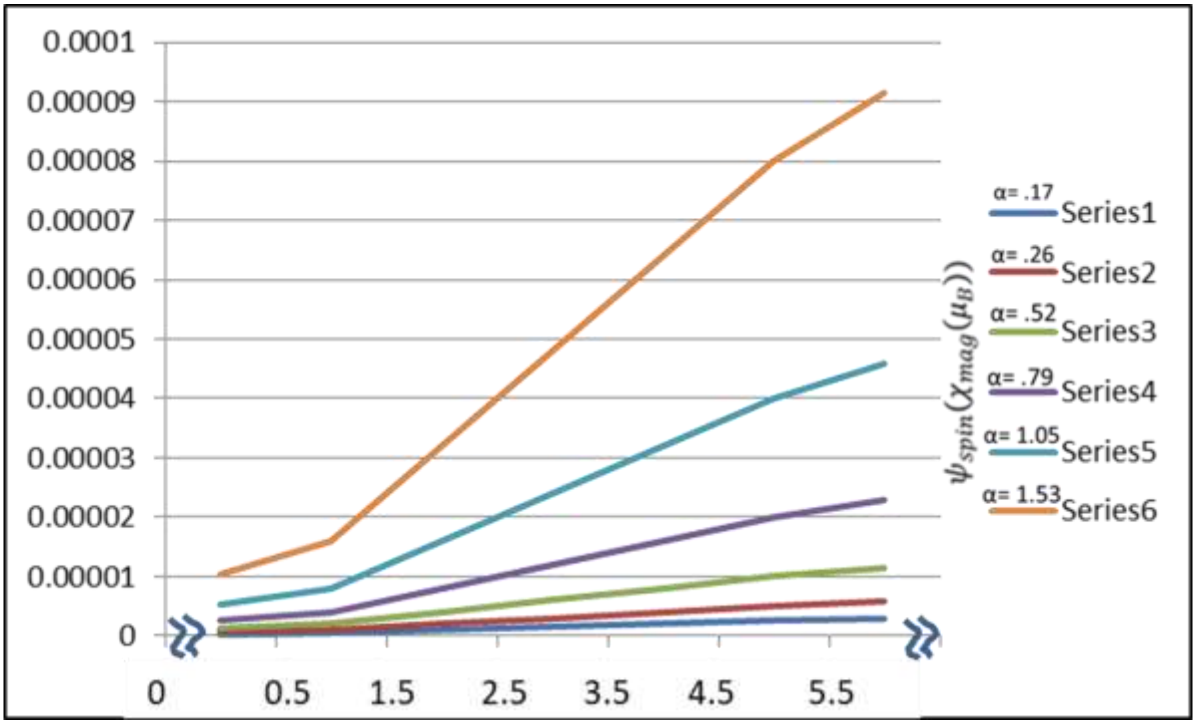

Figure:2. represents the spin eigenfunction at $0.5 \leq \left(n+\frac{1}{2}\right) \leq 5.5$ with $\frac{e^2}{\hbar c} \approx \frac{1}{137}$ for $\bar{J}\,\bar{S}$ coupling but with starching, twisting and twigging behavior ($0.17 \leq \alpha \leq 1.53\, rad$) and μ_B, ($58\ x\ 10^{-6} \frac{eV}{Tesla}$).

Figure:2. Shows energy field fluids of spinning electrons with interactions having $\frac{e^2}{\hbar c} \approx \frac{1}{137}$ and indeed $\bar{J}\,\bar{S}$ coupling, however with twisting and twigging in braided energy field configurations at integer fractional quantum states $0.5 \leq \left(n+\frac{1}{2}\right) \leq 5.5$, again preferably due to 'quantum tunneling' and 'quantum fluctuations (topological phases due to knotted spinning energy fields)' with QED [6].

On comparison of figures (1) and (2), we can say that electrons are not point particles but equivalent to diverse categories of energy fields fluids due to multiple 'quantum fluctuations', (phase changes) in space-time, therefore, quantum field theory is the most suitable option, of course, with QED behavior. We also observe the shape of the eigenfunction in both figures (1) and (2), with hyperbolic profile which is indicative of the fact that we ascribes the 'particle or quanta' with rest mass of the energy field fluids observed by detectors.

Our studies show that the whirlpools and the swirling potential barriers with various sizes are manifestations of fractional quantum states of the 'Quantum Field Theory, (QFT)' in topological space-time and of integer fractional quantum states of the QFT for 'spin dynamics and their interactions', in normal space-time. Physical constants are no more constant in topological space-time and even in QFT due to overwhelming quantum tunneling and quantum fluctuations (topological phases) following the quantum electrodynamics (QED) theory [6].

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