d-DISTANCE IN GRAPHS

----

*T.Jackuline , **Dr.J.Golden Ebenezer Jebamani ,***Dr. D. Premalatha

* Department of Mathematics, Sarah Tucker College and Research scholar (19221172092002) of PG & Research Department of Mathematics, Rani Anna Government College for Women, Manonmaniam Sundaranar University, Tamilnadu, India.

**Head & Assistant Professor, Department of Mathematics , Sarah Tucker College, Manonmaniam Sundaranar University, Tamilnadu, India.

***Head & Associate Professor, PG & Research Department of Mathematics, Rani Anna Government College for Women , Manonmaniam Sundaranar University, Tamilnadu, India.

Abstract: For two vertices u and v of a graph G, the usual distance d(u, v), is the length of the shortest path between u and v. In this paper we introduced the concept of d^d-distance by considering the degrees of various vertices presented in the path, in addition to the length of the path. We study some properties with this new distance. We define the eccentricities of vertices, radius and diameter of G with respect to the d^d-distance. First we prove that the new distance is a metric on the set of vertices of G. We compare the usual, geodesic and d^d-distances of two vertices u, v of V.

Keywords: Geodesic distance, d^d-distance, d^d-Eccentricity, d^d-Radius and d^d-Diameter

1. Introduction

By a graph G, we mean a non-trivial finite undirected connected graph without multiple edges and loops. Following standard notations (for any unexplained notation and terminology we refer [2]) V (G) or V is the vertex set of G and E(G) or E is the edge set of G = G(V, E). Let u, v be two vertices of G. The standard or usual distance d(u, v) between u and v is the length of the shortest u − v path in G. Chartrand el al [3] introduced the concept of detour distance in graphs as follows: For two vertices u, v in a graph G, the detour distance D(u, v) is defined as the length of the longest u − v path in G. In this article we introduce a new distance, which we call as Dd-distance between any two vertices of a graph G, and study some of its properties. This distance is significantly different from other distances. In some of the earlier distances, only path length was considered. Here we, in addition, consider the degree of u and v vertices present in a u − v path while defining its length. Using this length we define the Dd-distance. Chartand et al introduced the concept of detour distance by considering the length of the longest path between u and v. Kathiresan et al introduced the concept of superior distance and signal distance. In some of these distances only the lengths of various paths were considered.

In this chapter, the concept of *d*-distance in a connected graph *G* is introduced.

2. *d*-DISTANCE IN GRAPHS

Definition 2.1

Let u, v be two vertices of a connected graph G. Then the *d-length* of a u-v path defined as$d^d(u, v) = d(u, v) + \deg(u) + \deg(v) + \deg(u)\deg(v)$, where $d(u, v)$ is the shortest distance between the vertices u and v. The *d-distance* between two vertices u and v is defined as the d- length of a u-v path.

Example 2.2

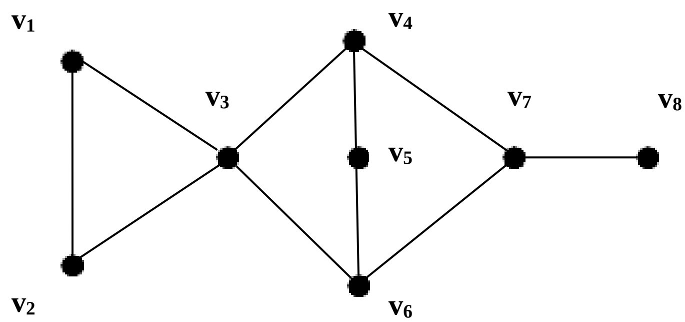

Figure 1: A graph *G* for *d*-distance between vertices

In figure , if $u = v_1$, $v = v_4$, then $d^d(u, v) = d(u, v) + \deg(u) + \deg(v) + \deg(u)\deg(v) = 2 + 2 + 3 + (2)(3) = 13$.

Definition 2.3 The d^d -eccentricity of any vertex v, $e^{dd}(v)$, is defined as the maximum distance from v to any other vertex, i.e., $e^{dd}(v) = \max\{ d^d(u, v) : u \in V(G)\}$

Definition 2.4 Any vertex u for which $d^d(u, v) = e^{dd}(v)$ is called d^d -eccentric vertex of v . Further, a vertex u is said to be d^d -eccentric vertex of G if it is the d^d -eccentric vertex of some vertex.

Definition 2.5 The d^d -radius, denoted by $r^{dd}(G)$, is the minimum d^d -eccentricity among all vertices of G i.e., $r^{dd}(G) = \min\{e^{dd}(v) : v \in V(G)\}$. Similarly the d^d -diameter, $d^d(G)$, is the maximum d^d -eccentricity among all vertices of G.

Theorem 2.6

If *G* is any connected graph, then the d^d-distance is a metric on the set of vertices of *G*.

Proof: Let *G* be a connected graph and $u, v \in V(G)$. Then it is clear by definition that $d^d(u, v) \geq 0$ and $d^d(u, v) = 0$ which implies $u = v$. It is obvious that $d^d(u, v) = d^d(v, u)$. It can be proved that the distance d^d satisfies the triangle inequality.

Case(i): Let $u, v, w \in V(G)$. Let P and Q be $u - w$ and $w - v$ shortest paths of shortest lengths $l(P)$ and $l(Q)$ respectively in G. Then $d^d(u, w) = l(P) + \deg(u) + \deg(w) + \deg(u)\deg(w)$ and $d^d(w, v) = l(Q) + \deg(w) + \deg(v) + \deg(w)\deg(v)$. Let $R = P \cup Q$ be the $u - v$ shortest path obtained by joining P and Q at w of length greater than one. Then

$$d^d(u, w) + d^d(w, v) = [l(P) + \deg(u) + \deg(w) + \deg(u)\deg(w)] + [l(Q) + \deg(w) + \deg(v) + \deg(w)\deg(v)]$$

$$= l(P \cup Q) + \deg(u) + \deg(v) + 2\deg(w) + \deg(u)\deg(w) + \deg(w)\deg(v)$$

$$= l(R) + \deg(u) + \deg(v) + 2\deg(w) + \deg(u)\deg(w) + \deg(w)\deg(v)$$

$$> l(R) + \deg(u) + \deg(v) + \deg(u)\deg(w) + \deg(w)\deg(v)$$

$$= d^d(u, v).$$

Therefore $d^d(u, w) + d^d(w, v) > d^d(u, v)$.

Case(ii): Suppose $R = P \cup Q$ be the $u - v$ shortest path of length one. Then we take a vertex w other than u and v and so we have either $u = w$ or $v = w$.

Subcase(i): Suppose that $u = w$. Then $d^d(u, w) = 0$.

Therefore $d^d(u, w) + d^d(w, v) = 0 + l(Q) + \deg(w) + \deg(v) + \deg(w)\deg(v) = l(P \cup Q) + \deg(u) + \deg(v) + \deg(u)\deg(v)$.

$d^d(u, w) + d^d(w, v) = d^d(u, v)$.

Subcase(ii): Suppose that $v = w$. Proof is similar to subcase(i).

Thus the triangular inequalities hold and hence d^d is a metric on the vertex set. ■

Corollary 2.7

For any three vertices u, v, w of a graph G, $d^d(u, v) \leq d^d(u, w) + d^d(w, v) - 2\deg(w)$

Proof: Let G be a connected graph.

$d^d(u, w) + d^d(w, v) = l(P \cup Q) + \sum_{u,v=x\epsilon P \cup Q} deg(x) + 2 \deg(w) + \sum_{u,v=x\epsilon P \cup Q} deg(x) + 2 \deg(w) + \prod_{u,=x\epsilon P \cup Q} deg(x) \geq d^d(u, v) + 2\deg(w)$.

$d^d(u, w) + d^d(w, v) \geq d^d(u, v) + 2\deg(w)$.

Hence $d^d(u, v) \leq d^d(u, w) + d^d(w, v) - 2\deg(w)$. ■

Proposition 2.8

If G is a connected graph with two distinct vertices u, v are adjacent then $d^d(u, v) \leq (n-1)(n+2)$.

Proof: Let u and v be two adjacent vertices. This implies that $d(u, v) \leq n-1$. By definition, $d^d(u, v) = d(u, v) + \deg(u) + \deg(v) + \deg(u)\deg(v) \leq \deg(u) + \deg(v) + (n-1) + \deg(u)\deg(v)$. Since $d(u, v) \leq n-1$, $\deg(v) \leq n-1$ for any vertices u and v. Hence $d^d(u, v) \leq (n-1)(n+2)$. ■

Theorem 2.9

For every tree $u, v \epsilon T$, $d^d(u, v) > d(u, v)$.

Proof: In a tree $d^d(u, v) = d(u, v)$. We know that $d^d(u, v) = d(u, v) + \deg(u) + \deg(v) + \deg(u)\deg(v)$

This implies that $d^d(u, v) > d(u, v)$. ■

The following results are immediate for a cycle and complete graph.

Theorem 2.10

(i). For a complete graph K_n, $d^d(u, v) = n^2$ for every $u, v \in K_n$.

(iii). For a complete bipartite graph $G = K_{m,n}$, $m<n$ and $V_m \cup V_n = V(G)$, $d^d(u, v) = n^2 + 2n + 2 \ \forall u, v \in V_m$.

(iii). For a star graph $G = K_{1,n}$, $d^d(u, v) = 2(n + 1) \forall v \in V(G)$

Proof: Above results are obvious by definitions. ■

Observation 2.11

For any connected graph G, if u and v are end vertices of G and $d(u, v) \leq n-1$ then $d^d(u, v) \leq (n-1)(n+2)$.

Conclusion:

Many researchers are concentrating various distance concepts in graphs. In this paper we have studied about d-distance in graphs. Then we have defined d^d -eccentricity , d^d -radius and d^d -diameter Many results have been found.

References

[1] F. Buckley and F. Harary, Distance in graphs, Addison- Wesley, Longman, 1990.

[2] G. Chartrand, H. Escuadro and P. Zhang, Detour distance in graphs, J. Combin. Comput, 53(2005) 75-94.

[3] G. Chartrand and P. Zang, Introduction to Graph Theory, Tata McGraw-Hill, (2006). [5] K.M. Kathiresan and G.Marimuthu, Superior distance in graphs, J. Combin. Comput., 61(2007) 73-80.

[4] K. M. Kathiresan and R. Sumathi, A study on signal distance in graphs, Algebra, graph theory and their applications, Narosa publishing house Pvt. Ltd(2010) 50-54.

[5] D. Reddy Babu and L. N. Varma, P., D-Distance in graphs, Golden Research Thoughts, 9(2013) 2231- 5063.

[6] Galeana-Snchez H, Xueliang Li, 1998, *Semikernals and* (*k, l*) - *kernals in digraphs,* SIAM J. Discrete Math, Vol. **11**, No. 2, 340-346.

[7] Haynes T. W, Hedetniemi S.T, Slater P. J, 1998, *Fundamentals of Domination in Graphs*, Marcel Dekker, Inc. New York.

[8] Haynes T. W, Hedetniemi S.T, Slater P. J, 1998, *Domination in Graphs: Advanced Topics*,

Marcel Dekker, New York.

[9] Lee C, 1994, *On the Domination Number of a Digraph*, Ph.D. Dissertation, Michigan State University.

[10] Włoch A, 2012, *On* 2-*dominating kernels in graphs*, Austr.. J. Combin. **53**: 273 - 284.

[11] Włoch A, Włoch I, 2009, *On* (k; 1) - *kernels in the corona of digraphs*, Int. J. Pure Appl. Math. **53** (4): 571 - 582.