Fuzzy Normed Near-rings and its Ideals

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Abstract- In this article we define a norm on a near-ring in order to present the idea of fuzzy normed near-rings for the first time and study some of its interesting properties. We prove the validity of certain results related to this structure. We also construct the ideals and study its behavior by establishing valuable conclusions.

***Index Terms*: :** Near-ring; Fuzzification; Normed Near-ring; R-subgroup; Idempotent t-Norm

INTRODUCTION

The notion "fuzzy sub near-ring " is first investigated by S. A. Zaid [1]. He introduced its ideals and established various results related to these structures. Later, many prominent researchers including S. D. Kim [7] and S. M. Hong [6] extended this idea and developed more constructions and results. Mark Farag and Brink van der Merwe [4] defined normed near-rings during their study of Lipschitz functions and near-ring related to it. Aykut Emniyet and Memet Sahin [3] used archimedean strict T-norm and archimedean strict T-conorm for the fuzification of normed rings. Their key constructions are normed fuzzy ring, normed fuzzy ideals, normed fuzzy prime and maximal ideals of normed fuzzy ring. They also proved that some of the algebraic properties of normed rings are valid in fuzzy normed rings.

Our main purpose of writing this article is to boost the process of fuzzification of well known algebraic structures. We introduce the idea of fuzzy normed near-ring and its associated fuzzy ideals by using idempotent T-norm and conorm. We explore various basic properties and results of this structure.

~~I.~~

I. MATERIALS AND METHODS

2.1Grassmannian Complex

This section is devoted to present some basic concepts and definitions which are essential to understand this work fully. This is to be noted that we label $\mathfrak{R}$ as a left near ring , Υ as the interval [0,1] and $\mathbb{G}$ as a group throughout the work.

Definition 1: A set $\mathfrak{R}$ possessing operations of addition and $\times$ constitutes a left near ring if $\forall \quad a,b,c \in \mathfrak{R}$ [leftmargin=*,labelsep=4.9mm]

1. $(\mathfrak{R},+)$ constitutes a group.
2. $(\mathfrak{R},.)$ forms semi group.
3. $a(b+c)=ab+ac$

Definition 2: [M. Akram (2007), M. M. Gupta (1991)] We call the mapping $T:\Upsilon\times\Upsilon\to\Upsilon$ a t-norm if $\forall \quad u,v,w\in\Upsilon$ [leftmargin=*,labelsep=4.9mm]

1. T is both commutative and associative.
2. $T(u',1)=u'$
3. $T(u',v')\leq T(u',w')$ if $v'\leq w'$

Definition 3: A mapping $T^*:\Upsilon\times\Upsilon\to\Upsilon$ is called an t-conorm if $\forall \quad u',v',w'\in\Upsilon$

1. T^* is both commutative and associative.
2. $T^*(u',0)=u'$
3. $T^*(u',v')\leq T^*(u',w')$ if $v'\leq w'$

Here we introduce a variant of t-norm and t-conorm by inserting an extra axiom of idempotency. We call these variants the idempotent t-norm and idempotent t-conorm.

Definition 4: [2, 5,10] A mapping $\Gamma:\Upsilon\times\Upsilon\to\Upsilon$ is called an idempotent t-norm if $\forall \quad u',v',w'\in\Upsilon$

1. Γ is commutative and associative.
2. $\Gamma(u',1)=u'$
3. $\Gamma(u',v')\leq\Gamma(u',w')$ if $v'\leq w'$
4. $\Gamma(u',u')=u'$

Similarly we can define idempotent t-conorm $\circ:\Upsilon\times\Upsilon\to\Upsilon$ as a conorm with idenpotent property. i.e.

$$\circ(u',u')=u \quad \forall \quad u'\in\Upsilon$$

Definition 5: [4,11] $\mathfrak{R}$ is referred as a normed near-ring if $\mathfrak{R}$ possesses a functional or norm $\|.\|:\mathfrak{R}\to\mathfrak{R}$ such that for $\nu,\omega\in\mathfrak{R}$ [leftmargin=*,labelsep=4.9mm]

1. $\|\nu\|=0\Leftarrow\nu=0$
2. $\|\nu+\omega\|\leq\|\nu\|+\|\omega\|$
3. $\|-\nu\|=\|-\nu\|$
4. $\|\nu\omega\|\leq\|\nu\| \quad \|\omega\|$

Definition 6: If ϱ denotes a subset of a set χ then the collection of possible tuples of the form $\{(p',\mu(p'))|p'\in\chi\}$ is termed as fuzzy subset of χ where $\mu:\chi\to[0,1]$.

Definition 7: Consider ϱ, $\mathcal{G}$ as fuzzy sets in a set χ, then the membership functions of their union, intersection and complement are given as under

$$(\varrho\cup\mathcal{G})(p')=Max\{\varrho(p'),\mathcal{G}(p')\}$$
$$(\varrho\cap\mathcal{G})(p')=Min\{\varrho(p'),\mathcal{G}(p')\}$$
$$\bar{\varrho}(p')=1-\varrho(p')$$

Definition 8: [1] Let the subset ϱ is fuzzy in a group $(\mathbb{G},+)$ then ϱ is said to be fuzzy sub-group in $\mathbb{G}$ if [leftmargin=*,labelsep=4.9mm]

1. $\varrho(p'-q')\geq Min\{\varrho(p'),\varrho(q')\};\forall \quad p',q'\in G$
2. $\varrho(-p')=\varrho(p');\forall \quad p'\in G$

Definition 9: [1] Let the subset ϱ is fuzzy in $(\mathbb{G},+)$ then ϱ is termed as fuzzy normal subgroup of $\mathbb{G}$ if [leftmargin=*,labelsep=4.9mm]

1. ϱ is fuzzy subgroup of $\mathbb{G}$.
2. $\varrho(b'+a'-b')=\varrho(a') \quad ;\forall a',b'\in G$

Definition 10:[1] Consider ϱ as fuzzy sub-set of $(\mathfrak{R}\ ,+\ ,.)$ then it becomes a fuzzy sub near-ring of $\mathfrak{R}$ if $\forall p',q'\in\mathfrak{R}$, we have , $\varrho(p'-q')\geq Min\{\varrho(p'),\varrho(q')\}$ and $\varrho(p'q')\geq Min\{\varrho(p'),\varrho(q')\}$. Morover, ϱ is said to be a fuzzy ideal (left , right) of near ring $\mathfrak{R}$ iff.

1. $(\varrho,+)$ constitute a fuzzy normal sub-group of $\mathbb{G}$.
2. $\varrho(p'q')\geq\varrho(q') \quad ;\forall p',q'\in\mathbb{G}$
3. $\varrho((p'+i)q'-p'q')\geq\varrho(i) \quad ;\forall p',q',i\in\mathbb{G}$

Next, we can define the operations sum, difference and product of the fuzzy sub near rings with the help of norms and conorms. For this, consider $F(R)$ to be the compilation of all fuzzy subsets of a near-ring $\mathfrak{R}$ and $\varrho, \vartheta \in F(\mathfrak{R})$, then we obtain the following

$$(\varrho+\vartheta)(z') = \circ\{\varrho(x') \star \vartheta(x')\}$$
$$(\varrho-\vartheta)(z') = \circ\{\varrho(x') \star \vartheta(x')\}$$
$$(\varrho\vartheta)(z') = \circ\{\varrho(x') \star \vartheta(x')\}$$

Definition 11: [1] A fuzzy sub-set ϱ of $\mathfrak{R}$ is termed as fuzzy $\mathfrak{R}$-subgroup of $\mathfrak{R}$ if $\forall\ a', y' \in \mathfrak{R}$

1. ϱ is fuzzy subgroup of ($\mathfrak{R}$,+)
2. $\varrho(a'y') \geq \varrho(y')$ and $\varrho(y'a') \geq \varrho(y')$

Now we proceed to introduce the normed version of the concepts mentioned above. That is, we present the normed version of fuzzy sub near-rings, fuzzy normal subgroup, fuzzy ideals and fuzzy R-subgroups. Which are defined as under.

Definition 12: A fuzzy subset ϱ, together with membership function $\varrho(x')$, of a fuzzy normed near-ring $\mathfrak{R}$ is termed as fuzzy normed sub near-ring if ϱ agrees with the conditions

1. $\varrho(p'-q') \geq \varrho(q') \star \varrho(q')$
2. $\varrho(p'q') \geq \varrho(p') \star \varrho(q')$

Definition 13: Suppose that ϱ_1 and ϱ_2 be two fuzzy normed sub near-rings over a normed near-ring $\mathfrak{R}$. We say ϱ_1 a fuzzy normed sub near-ring of ϱ_2 if $\forall a' \in \mathfrak{R}\quad \varrho_1(a') \leq \varrho_2(a')$

Definition 14: A fuzzy sub-set ϱ of a normed near-ring $\mathfrak{R}$ is labeled as fuzzy ideal of $\mathfrak{R}$ if $\forall p', q', x' \in \mathfrak{R}$

1. $\varrho(p'-q') \geq \varrho(p') \star \varrho(q')$
2. $\varrho(p'+x'-p') = \varrho(x')$
3. $\varrho(p'q') \geq \varrho(p')$
4. $\varrho\{(p'+x')q'-p'q'\} \geq \varrho(x')$

Definition 15: A sub-set ϱ of $\mathfrak{R}$ is termed as fuzzy normed $\mathfrak{R}$-subgroup of $\mathfrak{R}$ if $\forall a', b', y' \in \mathfrak{R}$

1. ϱ is fuzzy subgroup of ($\mathfrak{R}$,+).
2. $\varrho(a'y') \geq \varrho(y')$ and $\varrho(y'a') \geq \varrho(y')$

Definition 16: Let ϱ be a non constant fuzzy normed ideal of a normed near-ring $\mathfrak{R}$ and $\mathbb{B}_i$ be all other normed ideals of $\mathfrak{R}$ such that $\varrho \subseteq \mathbb{B}_i$ implies $\varrho^0 = \mathbb{B}_i^0$ or $\mathbb{B}_i = \lambda_{\mathfrak{R}}$, such an ideal ϱ is known as fuzzy normed maximal ideal of $\mathfrak{R}$.

II. RESULTS AND DISCUSSION

We devote this section to write the key findings related to $\mathfrak{R}$, ideals associated to $\mathfrak{R}$ and fuzzy normed $\mathfrak{R}$ -subgroup etc.

Lemma 1: For any fuzzy normed subnear-ring ϱ of a normed near-ring $\mathfrak{R}$ and $x' \in \mathfrak{R}$

(i) $\varrho(x') \leq \varrho(x')$ (ii) $\varrho(x') = \varrho(-x')$

Proof. Clear.

Proposition 1: If ϱ is afuzzy normed sub near-ring(ideal) of a normed near-ring $\mathfrak{R}$ then so is the set $\varrho_0 = \{y' \in \mathfrak{R}: \varrho(y') = \varrho(0)\}$.

Proof. Take $u, v \in \varrho_0 \Rightarrow \varrho(u') = \varrho(v') = \varrho(0)$.

Since ϱ is a fuzzy normed near-ring so

$$\varrho(u'-v') \geq \varrho(u') \star \varrho(v') = \varrho(0) \star \varrho(0) = \varrho(0)$$

On the other hand $\varrho(u'-v') \leq \varrho(0)$. This implies $\varrho(u'-v') = A(0)$ and therefore $u'-v' \in \varrho_0$. Similarly we can show that for $u', v' \in \varrho_0 \Rightarrow u'.v' \in \varrho_0$.

The sub-set $\mathcal{G}$ of a fuzzy normed near-ring is itself a fuzzy normed sub near-ring $\Leftarrow\ \mathcal{G}-\mathcal{G} \subseteq \mathcal{G}$ and $\mathcal{G}.\mathcal{G} \subseteq \mathcal{G}$.

Proof. First we suppose that $\mathcal{G}$ is fuzzy normed sub near-ring of $\mathfrak{R}$.

$\Rightarrow \mathcal{G}$ must be a fuzzy group with respect to $+$ and hence $\mathcal{G}-\mathcal{G} \subseteq \mathcal{G}$. Also for each $z' \in \mathfrak{R}$,

$$(\mathcal{G}.\mathcal{G})(z') = \underset{x'y'=z'}{\circ}(\mathcal{G}(x') \star \mathcal{G}(y')) \leq \underset{x'y'=z'}{\circ}\mathcal{G}(x'y')$$
$$= \mathcal{G}(z') \Rightarrow \mathcal{G}.\mathcal{G} \subseteq \mathcal{G}$$

Conversely, suppose that $\mathcal{G}-\mathcal{G} \subseteq \mathcal{G}$ and $\mathcal{G}.\mathcal{G} \subseteq \mathcal{G}$ then $\forall a', b' \in \mathfrak{R}$, we have

$$\mathcal{G}(a'-b') \geq (\mathcal{G}-\mathcal{G})(a'-b')$$
$$= \underset{a'-b'=p'-q'}{\circ}(A(p') \star \mathcal{G}(q'))$$
$$\geq \mathcal{G}(a') \star \mathcal{G}(b')$$

Similarly,

$$\mathcal{G}(a'b') \geq (\mathcal{G}\mathcal{G})(a'b') = \underset{a'b'=p'q'}{\circ}(\mathcal{G}(p') \star \mathcal{G}(q')) \geq$$

$\mathcal{G}(a') \star \mathcal{G}(b') \Rightarrow \mathcal{G}$ is fuzzy normed sub near-ring of $\mathfrak{R}$.

Suppose ϱ and $\mathcal{G}$ be fuzzy normed sub near-rings of fuzzy normed near-rings $\mathfrak{R}$ and $\mathcal{G}$ respectively. Also suppose $f: \mathfrak{R} \to \mathcal{G}$ near-ring homomorphism. Then

(i) $f(\varrho)$ is fuzzy normed subnear-ring of $\mathcal{G}$.

(ii) $f^{-1}(\mathcal{G})$ is fuzzy normed subnear-ring of $\mathfrak{R}$.

Proof. Let $r', s' \in \mathcal{G}$ and since f is surjective so there must be $p', q' \in \mathfrak{R}$ so that $f(p') = r'$ and $f(q') = s'$. also

$$(f(\varrho))(r') \star (f(\varrho))(s'$$
$$= \left(\underset{f(p')=r'}{\circ}(\varrho)(p')\right) \star \left(\underset{f(q')=s'}{\circ}(\varrho)(q')\right)$$
$$= \underset{f(p')=r', f(q')=s'}{\circ}(\varrho(p') \star \varrho(q'))$$
$$\leq \underset{f(p')=r', f(q')=s'}{\circ}(\varrho(p'-q'))$$
$$\leq \underset{f(p')-f(q')=r'-s'}{\circ}(\varrho(p'-q'))$$
$$= \underset{f(p'-q')=r'-s'}{\circ}(\varrho(p'-q'))$$
$$= \underset{f(x')=r'-s'}{\circ}(\varrho(x'))$$
$$= (f(\varrho))(r'-s')$$

In a same manner we can easily prove

$$(f(\varrho))(r'.s') \geq (f(\varrho))(r') \star (f(\varrho))(s')$$

(ii) This result can easily be obtain using the algorithm of (i).

Intersection of two fuzzy normed sub near-rings of $\mathfrak{R}$ is again a fuzzy normed fuzzy sub near-ring.

Proof. Take ϱ and $\mathcal{G}$ two fuzzy normed subnear-rings of $\mathfrak{R}$. Then $\forall a', a'' \in \mathfrak{R}$

$$(\varrho \cap \mathcal{G})(a'-a'') = min\{\varrho(a'-a''), \mathcal{G}(a'-a'')\}$$
$$\geq min(\varrho(a') \star \varrho(a''), \mathcal{G}(a') \star \mathcal{G}(a''))$$
$$\geq min((\varrho \cap \mathcal{G})(a'), (\varrho \cap \mathcal{G})(a''))$$

similarly,

$$(\varrho \cap \mathcal{G})(a'.a'') = min\{\varrho(a'.a''), \mathcal{G}(a'.a'')\}$$
$$\geq min\{\varrho(a') \star \varrho(a''), \mathcal{G}(a') \star \mathcal{G}(a'')\}$$
$$\geq min\{(\varrho \cap \mathcal{G})(a'), (\varrho \cap \mathcal{G})(a'')\}$$

A normed near-ring can not be the union of two proper fuzzy normed subnear-rings.

Proof. Let ϱ , $\mathcal{G}$ be two fuzzy normed proper subnear-rings of $\mathfrak{R}$ such that $\varrho(x') = 1$ or $\mathcal{G}(x') = 1$ for all $x' \in \mathfrak{R}$. Also suppose $a', b' \in \mathfrak{R}$ with $\varrho(a') = 1 = \mathcal{G}(b')$ and $\varrho(b') \leq 1, \mathcal{G}(a') \leq 1$.

Now, if $\varrho(a'b') = 1$ then since $\varrho((a')^{-1}) = 1$ so we have

$$\varrho(b') = \varrho((a')^{-1}(a'b')) \geq \varrho((a')^{-1}) \star \varrho(a'b') = 1 \Rightarrow \varrho(b') \geq 1.$$

Which is a contradiction and similar contradiction can also be obtain for the case $\mathcal{G}(a'b') = 1$. Hence ϱ and $\mathcal{G}$ cannot be proper.

Let ϱ be a fuzzy normed ideal of $\mathfrak{R}$ then

$$\varrho(a' - b') = \varrho(0) \Rightarrow \varrho(a') = \varrho(b')$$

Proof. Assume that ϱ be fuzzy normed ideal of $\mathfrak{R}$ and $\forall a', a'' \in \mathfrak{R}$ we have $\varrho(a' - a'') = \varrho(0)$. Then

$$\varrho(a') = \varrho(a' - a'' + a'') \geq \varrho(a' - a'') \star \varrho(a'') = \varrho(0) \star \varrho(a'') = \varrho(a'')$$

$$\varrho(a') \geq \varrho(a'')$$

also

$$\varrho(a'') = \varrho(a'' - a' + a') \geq \varrho(a'' - a') \star \varrho(a') = \varrho(0) \star \varrho(a') = \varrho(a')$$

$$\varrho(a'') \geq \varrho(a')$$

Hence $\varrho(a') = \varrho(a'')$

Intersection of two fuzzy normed (right, left) ideals of a normed near-ring $\mathfrak{R}$ constitutes a fuzzy normed (right, left) ideal of R.

Proof. Consider two fuzzy normed left ideals ϱ and $\mathcal{G}$ of $\mathfrak{R}$ then it is clear that they are fuzzy normed subnear-rings according to Theorem. 3 their intersection must be a fuzzy normed subnear-ring. Now for all $a', b', x' \in \mathfrak{R}$ we have

$$(\varrho \cap \mathcal{G})(a' + x' - a') = min\{\varrho(a' + x' - a'), \mathcal{G}(a' + x' - a')\}$$
$$\geq min(\varrho(x'), \mathcal{G}(x'))$$
$$\geq min((\varrho \cap \mathcal{G})(x'))$$

$\Rightarrow \varrho \cap \mathcal{G}$ is fuzzy normal subgroup. Again we have

$$(\varrho \cap \mathcal{G})(a'x') = min\{\varrho(a'x'), \mathcal{G}(a'x')\}$$
$$\geq min(\varrho(x'), \mathcal{G}(x'))$$
$$\geq min((\varrho \cap \mathcal{G})(x'))$$

$\Rightarrow \varrho \cap \mathcal{G}$ is fuzzy left ideal and hence $\varrho \cap \mathcal{G}$ is fuzzy normed ideal of $\mathfrak{R}$. Similarly one can obtain this result for right ideals.

Let ϱ exhibits a fuzzy right ideal of normed near-ring $\mathfrak{R}$. If we describe a fuzzy sub-set $\kappa\varrho$, for any $\kappa \in \mathfrak{R}$, as $\kappa\varrho(z') = \varrho(\kappa z')$, then $\kappa\varrho$ is fuzzy right ideal of R.

Proof. Take any $\kappa \in \mathfrak{R}$ then for all $a', a'' \in \mathfrak{R}$ we have

$$\kappa\varrho(a' - a'') = \varrho\{\kappa(a' - a'')\} = \varrho\{(\kappa a' - \kappa a'')\}$$
$$\geq \varrho\{\kappa a'\} \star \varrho\{\kappa a''\} = \kappa\varrho(a') \star \kappa\varrho(a'')$$

and

$$\kappa\varrho(a' + a'' - a') = \varrho\{\kappa(a' + a'' - a')\} = \varrho\{(\kappa a' + \kappa a'' - \kappa a')\}$$
$$= \varrho\{\kappa a''\} = \kappa\varrho(a'')$$

$\Rightarrow$ $\kappa\varrho$ is normal fuzzy subgroup.

Now,

$$\kappa\varrho\{(a' + y')a'' - a'a''\} = \varrho\{\kappa((a' + y')a'' - a'a'')\} = \varrho\{(\kappa a' + \kappa y')a'' - (\kappa a')a''\} \geq \varrho\{\kappa y'\} = \kappa\varrho(y')$$

Hence $\kappa\varrho$ is fuzzy right ideal of $\mathfrak{R}$.

The fuzzy ideals(left, right) of normed near-rings are nothing but constant functions.

Proof. Let ϱ be representing a fuzzy left ideal of normed near-ring $\mathfrak{R}$ and $a', b' \in \mathfrak{R}$ implies $\varrho(a'b') \geq \varrho(b')$. By substitution $b' = e$ we get $\varrho(a'e) = \varrho(a') \geq \varrho(e)$. But $b = (a')^{-1}$ gives us $\varrho(a'(a')^{-1}) = \varrho(e) \geq \varrho((a')^{-1}) = \varrho(a')$. Hence $\varrho(a') = \varrho(e) =$ Constant.

A subset ϱ of a normed near-ring $\mathfrak{R}$ constitutes an $\mathfrak{R}$-sub-group(two sided) of $\mathfrak{R}$ $\Leftarrow \lambda_\varrho$ is fuzzy normed $\mathfrak{R}$-sub-group(two sided) of $\mathfrak{R}$.

Proof. let ϱ be $\mathfrak{R}$-subgroup of $\mathfrak{R}$. Then for $a', b' \in \varrho$ and $r' \in \mathfrak{R} \Rightarrow a' - b', a'r', r'a' \in \varrho$.

Now for all $r', u', v' \in \mathfrak{R}$, we have two cases

(i) If $u', v' \in \varrho$ then we must have $u' - v', r'u' \in \varrho$ and so

$$\lambda_\varrho(u' - v') = 1 \geq \lambda_\varrho(u') \star \lambda_\varrho(v')$$
$$\lambda_\varrho(r'u') = 1 \geq \lambda_\varrho(u') \star \lambda_\varrho(v')$$

(ii) If atleast one of u' and v' is not in ϱ . Let $u' \notin \varrho$ then $\lambda_\varrho(u') = 0$ and therefore

$$\lambda_\varrho(u' - v') \geq \lambda_\varrho(u') \star \lambda_\varrho(v')$$

also if $u' \notin \varrho$ implies $\lambda_\varrho(u') = 0$ and therefore

$$\lambda_\varrho(r'u') \geq \lambda_\varrho(u') \quad \text{and} \quad \lambda_\varrho(u'r') \geq \lambda_\varrho(u')$$
$$\lambda_\varrho(r'u') \geq \lambda_\varrho(u') \star \lambda_\varrho(v')$$

Let ϱ is a fuzzy sub-set of a normed near-ring $\mathfrak{R}$ and $\mathfrak{R}_\varrho^\kappa$ is level subset of R. Then $\mathfrak{R}_\varrho^\kappa$ is normed subnear-ring of $\mathfrak{R}$ if ϱ is fuzzy normed subnear-ring(ideal) in $\mathfrak{R}$.

Proof. Let ϱ be fuzzy normed subnear-ring of $\mathfrak{R}$ and $a', a'' \in \mathfrak{R}_\varrho^\kappa$. This implies $\varrho(a') \geq \kappa$, $\varrho(a'') \geq \kappa$. also

$$\varrho(a' - a'') \geq \varrho(a') \star \varrho(a'') \geq \kappa \star \kappa \geq \kappa$$
$$\Rightarrow (a' - a'') \in \mathfrak{R}_\varrho^\kappa$$
$$\varrho(a'a'') \geq \varrho(a') \star \varrho(a'') \geq \kappa \star \kappa \geq \kappa$$
$$\Rightarrow a'.a'' \in \mathfrak{R}_\varrho^\kappa$$

$\Rightarrow$ $\mathfrak{R}_\varrho^\kappa$ is subnear-ring of R.

If ϱ be a normed fuzzy left maximal ideal of normed near-ring $\mathfrak{R}$, then $\varrho(0) = 1$

Proof. On contrary we suppose $\varrho(0) \neq 1$. Also let $1 > s' > \varrho(0)$ and for a fuzzy subset $\mathcal{G}$ of $\mathfrak{R}$ we have $\mathcal{G}(y') = s'$ for all $y' \in \mathfrak{R}$.

$\Rightarrow$ $\mathcal{G}$ is an ideal of $\mathfrak{R}$ as $\mathcal{G}$ behave like a constant function.

We can also conclude that $\varrho \subset \mathcal{G}$, $\mathcal{G} \neq \lambda_{\mathfrak{R}}$ and

$$\mathcal{G}^0 = \{y' \in \mathfrak{R} \ \mid \ \mathcal{G}(y') = \mathcal{G}(0)\} = \mathfrak{R}$$

Clearly these facts contradict the definition of the maximality of normed fuzzy ideal ϱ . Hence $\varrho(0) = 1$.

III. CONCLUSION

This This work shows that the fuzzification process can be extented to the concept normed near-rings and useful results related to fuzzy normed near-rings can be obtained.

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