EDGE SUM LABELING ON PENDENT GRAPHS

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Abstract

Let G (V, E) be a graph with p vertices and q edges. Let f:$V(G)$→{0,1,2,...,p-1} be a bijection such that the induced function f^*:$E(G)$→N defined by $f^*_{sum}(uv)$=$f(u)$+$f(v)$ for every uv∈$E(G)$. If f^*_{sum} is injective then f^*_{sum} is called an edge sum graph labeling of G. A graph which admits sum graph labeling is called edge sum graph.

1. Introduction

Every graph in this paper are simple finite, undirected and non-trivial graph $G(V, E)$ with vertex set V and Edge set E. For graph theoretic terminology we refer to Harary. R. Ponraj, J. Vijaya Xavier Parthipan and R. Kala defined pair sum labeling. We introduce edge sum labeling on pendent graphs

Definition 2.1

Pendent graph with a vertex is obtained by appending a pendent edges of each vertex of a path.

Definition 2.2

Pendent graph with 2 vertex is obtained by appending a pendent edges of each vertex of a path.

Definition 2.3

Pendent graph with 3 vertex is obtained by appending a pendent edges of each vertex of a path.

Theorem 3.1

Pendent graph with a pendent is an edge sum graph.

Proof:

Let V={$v_i/1 \leq i \leq n$} ∪{$u_i/1 \leq i \leq n$} be the vertex set of simple pendant graph with 1 pendant and E=$\bigcup_{i=1}^{2} E_i$

Let $E_1 = \{v_i v_{i+1}/\ 1 \leq i \leq n-1\}$

$E_2 = \{v_i u_i/\ 1 \leq i \leq n\}$

Here |V (G)|=2n

|E (G)|=2n-1

Define f:V→ {0, 1, 2,.. ,p-1} by

$$f(v_i) = \{2(i-1)/1 \le i \le n\}$$
$$f(u_i) = \{2i - 1/1 \le i \le n\}$$

Then the corresponding edge label are as follows

$f^*_{sum}(v_i v_{i+1}) = f(v_i) + f(v_{i+1})$

$=2(i-1) + 2(i+1-1)$

$= 2i - 2 + 2i$

$= 4i - 2$

$f^*_{sum}(v_i u_i) = f(v_i) + f(u_i)$

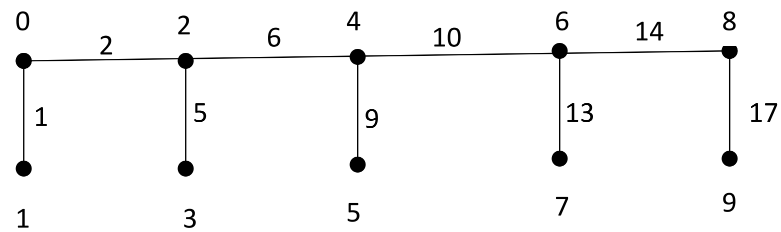
$=2(i-1) + 2i - 1$

$= 2i - 2 + 2i - 1$

$= 4i - 3$

Clearly the edge labels are distinct.

Therefore, pendant graph with a pendant is a edge sum graph.



Theorem 3.2

Pendant graph with 2 pendant is a edge sum graph.

Proof

Let $V = \{v_i \mid 1 \le i \le n\} \cup \{u_i \mid 1 \le i \le n\} \cup \{w_i \mid 1 \le i \le n\}$

be the vertex set of simple pendant graph with 2 pendant and let

E=$\cup_{i=1}^{3} E_i$

$E_1 = \{v_i v_{i+1} | 1 \le i \le n-1\}$

$E_2 = \{v_i u_i | 1 \le i \le n\}$

$E_3 = \{v_i w_i | 1 \le i \le n\}$

Here |V (G)|=3n and |E (G)|=3n-1

Define f:$V \rightarrow \{0,1,2,\dots,p-1\}$ by

$$f(v_i) = \{3(i-1) | 1 \le i \le n\}$$

$$f(u_i) = \{3i-2 | 1 \le i \le n\}$$

$$f(w_i) = \{3i-1 | 1 \le i \le n\}$$

Then the corresponding edge labeling is as follows

$$f^*_{sum}(v_i v_{i+1}) = f(v_i) + f(v_{i+1})$$

$$= 3(i-1) + 3(i+1-1)$$

$$= 3(i-1) + 3i$$

$$= 6i - 3$$

$$f^*_{sum}(v_i u_i) = f(v_i) + f(u_i)$$

$$= 3(i-1) + 3i - 2$$

$$= 3i - 3 + 3i - 2$$

$$= 6i - 5$$

$$f^*_{sum}(v_i w_i) = f(v_i) + f(w_i)$$

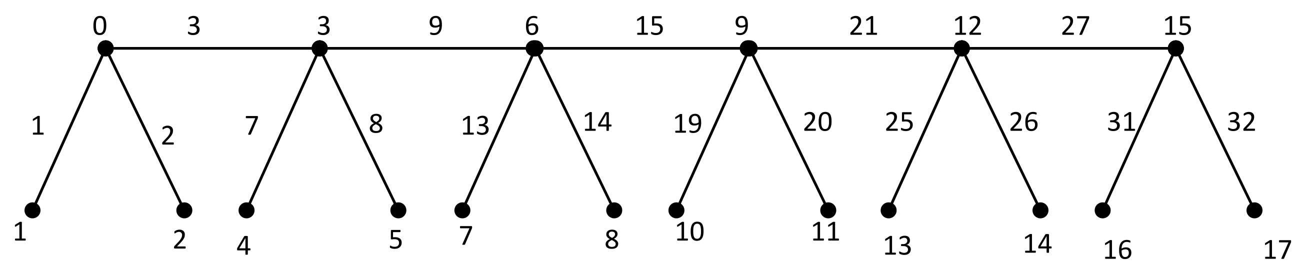
$$= 3(i-1) + 3i - 1$$

$$= 3i - 3 + 3i - 1$$

$$= 6i - 4$$

Clearly the edge labels are distinct.

Therefore pendant graph with 2 pendant is a edge sum graph.



Theorem 3.3

Pendant graph with 3 pendant is a edge sum graph.

Proof:

Let $V = \{v_i \mid 1 \le i \le n\} \cup \{u_i \mid 1 \le i \le n\} \cup \{w_i \mid 1 \le i \le n\} \cup \{x_i \mid 1 \le i \le n\}$

be the vertex set of pendant graph with 3 pendant and

$$E = \bigcup_{i=1}^{4} E_i$$

Let $E_1 = \{v_i v_{i+1} / \, 1 \leq i \leq n-1\}$

$E_2 = \{v_i u_i / \, 1 \leq i \leq n\}$

$E_3 = \{v_i w_i / \, 1 \leq i \leq n\}$

$E_4 = \{v_i x_i / \, 1 \leq i \leq n\}$

Here $|(G)|$=4n

$|(G)|$=4n-1

Define $f: V \rightarrow \{0,1,2, \dots, p-1\}$ by

$$f(v_i) = \{4(i-1) | 1 \leq i \leq n\}$$

$$f(u_i) = \{4i-3 | 1 \leq i \leq n\}$$

$$f(w_i) = \{4i-2 | 1 \leq i \leq n\}$$

$$f(x_i) = \{4i-1 | 1 \leq i \leq n\}$$

Then the corresponding edge labeling is as follows

$$f_{sum}^{*}(v_i v_{i+1}) = f(v_i) + f(v_{i+1})$$

$$= 4(i-1) + 4i$$

$$= 4i - 4 + 4i$$

$$= 8i - 4$$

$$f_{sum}^{*}(v_i u_i) = f(v_i) + f(u_i)$$

$$= 4(i-1) + (4i-3)$$

$$= 4i - 4 + 4i - 3$$

$$= 8i - 7$$

$$f_{sum}^{*}(v_i w_i) = f(v_i) + f(w_i)$$

$$= 4(i-1) + 4i - 2$$

$$= 4i - 4 + 4i - 2$$

$$= 8i - 6$$

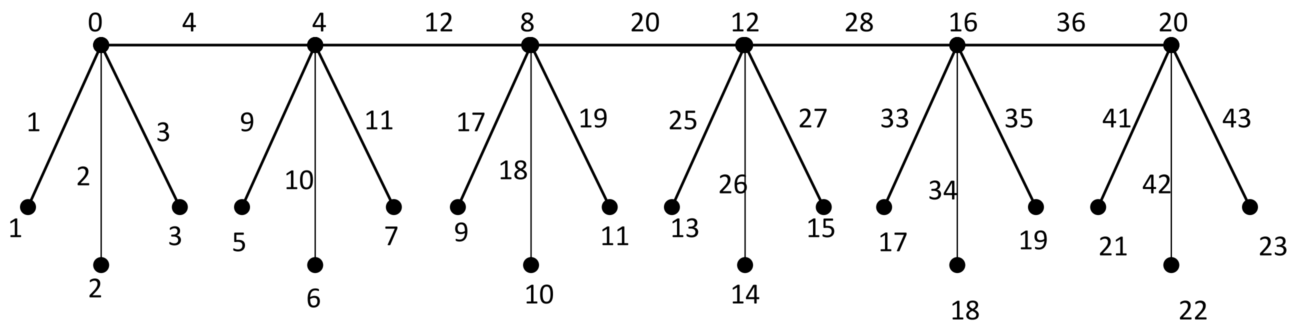
$$f_{sum}^{*}(v_i x_i) = f(v_i) + f(x_i)$$

$$= 4(i-1) + 4i - 1$$

$$= 4i - 4 + 4i - 1$$

$$= 8i - 5$$

Clearly the edge labels are distinct. Therefore pendant graph with 3 pendant is a edge sum graph.



4 Conclusions

In this paper, we proved Pendent graph with a vertex, Pendent graph with 2 vertices, Pendent graph with 3 vertices are edge sum graph. There may be interesting edge sum graphs can be constructed in future.

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