A unique hybrid approach for solving convection-diffusion problems

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Abstract: The core objective of this study is to develop a hybrid of homotopy perturbation transform method along with Atangana Baleanu Caputo operator for determination of numerous linear and nonlinear convection-diffusion problems arising in physical phenomena. The homotopy perturbation transform methodology is a bland of homotopy perturbation transform method and Laplace transform method. The nonlinear terms can be easily handled by the use of He's polynomials but we tried to utilize Atangana Baleanu Caputo operator (ABC-operator) to change the potency of conversion of series result. The approach helps in nursing improvement in the methodology to resolve many sorts of partial differential equations. The approximate solutions obtained by the projected theme in an exceedingly big selection of the problems domain were compared with those results obtained from the particular solutions. The comparison shows an explicit agreement between the results.

Keywords: Atangana Baleanu Caputo operator, Convection-diffusion, Homotopy perturbation transform method, Laplace transform.

Introduction

Recent studies in science and engineering incontestable that the dynamics of the many systems could also be represented with additional accuracy by employing that of differential equations of non-integer order. The diffusion equation may be a partial differential equation that portrays density dynamics in a very material subject to diffusion [1,2]. The convection-diffusion equation explains the owe of warmth, particles, oil reservoir simulations, transport of mass and energy, international weather production, or different physical quantities in conditions wherever their square measure each diffusion and convection or temperature change [3,4]. Down diffusion equations square measure mostly employed in describing abnormal slow diffusion development, and down diffusion equations square measure perpetually employed in describing abnormal convection development. Time-fractional diffusion comes by considering continuous-time stochastic process issues, that square measure normally Non-Markovian processes [5-7].
Several definitions, associated with three-quarter order-derivatives are employed in the literature. These definitions embody Riemann-Liouville, Liouville-Caputo, conformable derivatives, Caputo-Fabrizio, Atangana-Baleanu, and Atangana Koca, to say a couple of [8-11]. The selection of three-quarter differentiation is intended by the very fact that the interaction with the medium isn't native however world. The three-quarter operators will be a helpful thanks to

embody memory in a very dynamic method. A propellant method that's sculptural through fractional-order derivatives carries info regarding its gift in addition as past states [12-16].

In this paper, we have a tendency to think about the time-fractional diffusion and convection-diffusion equations, obtained from the quality equations by substitution the time spinoff with incomplete derivatives of kind Liouville-Caputo, Atangana-Baleanu-Caputo, incomplete conformable spinoff in Liouville-Caputo sense and Atangana-Koca-Caputo of order a, with 0 < a = 1 [17-21].

The convection-diffusion condition may be a blend of the dissemination and convection conditions and portrays physical wonders wherever particles, vitality, or distinctive physical amounts unit exchanged inside a physical framework owing to two processes diffusion and convection. at interims the ultimate kind, the convection-diffusion condition is given as follows [22-31].

$$v_t = \nabla.(D.\nabla v) - \nabla.(v) + Q \quad (1)$$

where v is that the variable of intrigued, D is that the diffusivity, such as mass diffusivity for molecule movement or warm diffusivity for heat transport, t is that the normal speed that the number is moving [32]. As a case, in advection, u may possibly be the concentration of salt in a surpassing waterway, at that point v_t would be the rate of the water stream [33,34].

He's convection-diffusion condition could be a blend of the dissemination and convection conditions and depicts physical wonders wherever particles, vitality, or elective physical quantities are exchanged inside a physical framework due to two processes: diffusion and convection. interior the in general sort, the convection-diffusion condition is given as follows where u is that the variable of intrigued, D is that the diffusivity, such as mass diffusivity for molecule movement or warm diffusivity for heat transport, t is that the common rate that the sum is moving [25,35-37]. As relate case, in temperature alter, u may conceivably be the concentration of salt in relate uncommon stream, at that point v_t would be the speed of the water stream [38,39].

The basic motivation of this study is to utilize a booming adjustment of HPTM to vanquish the lacking. we have an inclination to actualize the Homotopy irritation alteration method for tending to the convection-dispersion conditions. Utilizing this procedure, all conditions may be culminated. to boot, exceptionally exact comes about unit gotten in an exceedingly wide reach through a number of stretch steps [40-42]. The proposed hybrid offers the course of action in an exceedingly quick manner. The use of He's polynomials in nonlinear terms initially anticipated by Ghorbani [43]. Some models are given to affirm the immovable quality and viability of the HPTM.

Most of the contemplations of people have repaired close the progress of science and improvement, thus got to the plausibleness of the thoroughbred "consistent course of activity". Some individuals acknowledge that a shrewd verbalization of over 10 terms is very tangled and so is purposeless basically [44,45]. This was coordinated varied a protracted time back once there have been no PCs and people created a "consistent arrange" on paper and opt for it by hand. Around that time, it had been associate degree doubt a hard work to hit the books an illustrative phraseology of over ten terms. Our common place thought of "exact course of activity" was pictured in such quite condition, for all intents and functions. In any case, luckily, we have a tendency to area unit directly at intervals the hour of a laptop with monumental memory and quick central handling the unit it desires because it was variety of minutes to hit the books a coherent phraseology so with varied terms, a lot of snappier than 2 or 3 terms by hand [46]. Operator estimation programming, for the occasion, Mathematica, MatLab, Maple, etc, area unit connected broader and broader, and that is because it was the tip of the ice sheet, what is additional, additional investigators conclude their logical conditions on a private laptop while not papers and pens by any implies [47,48].

In these cases, paper and pen area unit outmoded by a tough plate and support of a laptop. Likewise, as of now, the web transforms into another notable medium, almost like the presence of the paper quite a long whereas earlier. The maker eventually acknowledges that the quality thought of Associate in Nursing "indicative plan" ought to be rested to travel up against the laptop time, and a coherent verbalization with numerous terms may well be recognized by most scientists and examiners shortly from currently [17]. By the day's finish, within the hour of the laptop, a descriptive rationalization is not wholly crucial to be solely a number of terms. At this time, the Homotopy examination procedure is for the hour of the laptop, much.

Occupations within the packaging of the Homotopy assessment strategy and unravel its applications in science and building associate degree unequivocal indicative game arrange is given on the grounds that, with algorithmic conditions for coefficients. This logical arrange agrees well with mathematical results and might be seen as associate degree importance of the course of action of the pondered nonlinear issue.

Fractionalized Governing Equations

Right now, adequacy and the helpfulness of Homotopy Perturbation Transform Method (HPTM) are shown by finding precise arrangement of Convection Diffusion Equation.

Example No. 1 Consider the following fractional Convection Diffusion Equation.

$$\frac{\partial u}{\partial t} = \frac{\partial^2 u}{\partial x^2} - u$$

$$u_t = u_{xx} - u \tag{2}$$

and subject to the constant initial conditions

$$u(x\,,0) = x + e^{-x}. \tag{3}$$

On both sides of Eqs. (2.1) we take Laplace transform by using Atangana-Baleanu-Caputo Operator then by using the Laplace transform of differential property

$$\mathcal{L}(u_t) = \mathcal{L}(u_{xx} - u)$$

$$S\mathcal{L}u(x,s) - u(x,0) = \frac{1}{a_o N(\propto)} \frac{s^\alpha + a_1}{s^\alpha} \mathcal{L}(u_{xx} - u)$$

$$\mathcal{L}u(x,s) = \frac{u(x,0)}{s} + \frac{1}{s}\left[\frac{1}{a_o N(\alpha)}\frac{s^\alpha + a_1}{s^\alpha}\mathcal{L}(u_{xx} - u)\right] \quad (4)$$

By applying the initial conditions (2.2), we will have

$$\mathcal{L}u(x,s) = \frac{x - e^{-x}}{s} + \frac{1}{s}\left[\frac{1}{a_o N(\alpha)}\frac{s^\alpha + a_1}{s^\alpha}\mathcal{L}(u_{xx} - u)\right] \quad (5)$$

On both sides of the Eqs. (2.4) applying inverse Laplace transform, we obtain

$$u(x,t) = \mathcal{L}^{-1}\left[\frac{x - e^{-x}}{s} + \left[\frac{1}{a_o N(\alpha)}\frac{s^\alpha + a_1}{s^{\alpha+1}}\mathcal{L}(u_{xx} - u)\right]\right] \quad (6)$$

$$u(x,t) = x + e^{-x} + \mathcal{L}^{-1}\left[\frac{1}{a_o N(\alpha) s}\mathcal{L}(u_{xx} - u) + \frac{a_1}{a_o N(\alpha) s^{\alpha+1}}\mathcal{L}(u_{xx} - u)\right] \quad (7)$$

By using HPTM the solutions of a function $u(x;t)$ is given by

$$u(x,t) = \sum_{n=0}^{\infty} p^n u_n(x,t) \quad (8)$$

Involving Eq. (2.8) in Eq. (2.6) and (2.7), we have

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = x + e^{-x} + p\mathcal{L}^{-1}\left[\left[\frac{1}{a_o N(\alpha) s}\mathcal{L}\{(\sum_{n=0}^{\infty} p^n u_n(x,t)_{xx}) - (\sum_{n=0}^{\infty} p^n u_n(x,t))\}\right] + \left[\frac{a_1}{a_o N(\alpha) s^{\alpha+1}}\mathcal{L}\{(\sum_{n=0}^{\infty} p^n u_n(x,t)_{xx}) - (\sum_{n=0}^{\infty} p^n u_n(x,t))\}\right]\right] \quad (9)$$

Now, by comparing the coefficients of identical power of P, we get

$For\ P_o$:

$$u_0(x,t) = x + e^{-x} \quad (10)$$

$For\ P_1$:

$$u_1(x,t) = \mathcal{L}^{-1}\left[\left[\frac{1}{a_o N(\alpha) s}\mathcal{L}\{(x + e^{-x})_{xx} - (x + e^{-x})\}\right] + \left[\frac{a_1}{a_o N(\alpha) s^{\alpha+1}}\mathcal{L}\{(x + e^{-x})_{xx} - (x + e^{-x})\}\right]\right]$$

$$u_1(x,t) = -\frac{xt}{a_o N(\alpha)} - \frac{a_1 x t^{\alpha+1}}{a_o N(\alpha)(\alpha+1)!} \quad (11)$$

Now, put the value of $N(\propto) = 1, a_1 = \frac{\alpha}{1-\alpha}$ and $a_o = \frac{1}{1-\alpha}$ in the above equation

$$u_1(x,t) = -\frac{xt(1-\alpha)}{(1)} - \frac{\alpha/_{1-\alpha}}{(1/_{\alpha-1})(1)} \times \frac{xt^{\propto+1}}{(\alpha+1)!} \quad (12)$$

Now, put $\alpha = 1$

$$u_1(x,t) = -\frac{xt^2}{2!} \quad (13)$$

$For\ P_2$:

$$u_2(x,t) = \mathcal{L}^{-1}\left[\left[\frac{1}{a_oN(\alpha)s}\mathcal{L}\left\{\left(-\frac{xt}{a_oN(\alpha)} - \frac{a_1xt^{\alpha+1}}{a_oN(\alpha)(\alpha+1)!}\right)_{xx} - \left(-\frac{xt}{a_oN(\alpha)} - \frac{a_1xt^{\alpha+1}}{a_oN(\alpha)(\alpha+1)!}\right)\right\}\right] + \left[\frac{a_1}{a_oN(\alpha)s^{\alpha+1}}\mathcal{L}\left\{\left(-\frac{xt}{a_oN(\alpha)} - \frac{a_1xt^{\alpha+1}}{a_oN(\alpha)(\alpha+1)!}\right)_{xx} - \left(-\frac{xt}{a_oN(\alpha)} - \frac{a_1xt^{\alpha+1}}{a_oN(\alpha)(\alpha+1)!}\right)\right\}\right]\right]$$

$$u_2(x,t) = \frac{xt^2}{a_o^2N^2(\alpha)2!} + \frac{a_1x\,t^{\alpha+2}}{a_o^2N^2(\alpha)(\alpha+2)!} + \frac{a_1x\,t^{\alpha+1}}{a_o^2N^2(\alpha)(\alpha+1)!} + \frac{{a_1}^2xt^{2\alpha+2}}{{a_o}^2N^2(\alpha)(2\alpha+2)} \tag{14}$$

Now, put the value of $N(\propto) = 1, a_1 = \frac{\alpha}{1-\alpha}$ and $a_o = \frac{1}{1-\alpha}$ in the above equation

$$u_2(x,t) = \frac{\alpha^2}{1-\alpha^2} \times (1-\alpha)^2 \times \frac{xt^{2\alpha+2}}{(2\alpha+2)!} \tag{15}$$

Now, put $\alpha = 1$

$$u_2(x,t) = \frac{xt^4}{4!} \tag{16}$$

$For\ P_3$:

$$u_3(x,t) = \mathcal{L}^{-1}\left[\left[\frac{1}{a_oN(\alpha)s}\mathcal{L}\left\{\left(\frac{xt^2}{a_o^2N^2(\alpha)2!} + \frac{a_1x\,t^{\alpha+2}}{a_o^2N^2(\alpha)(\alpha+2)!} + \frac{a_1x\,t^{\alpha+1}}{a_o^2N^2(\alpha)(\alpha+1)!} + \frac{{a_1}^2xt^{2\alpha+2}}{{a_o}^2N^2(\alpha)(2\alpha+2)}\right)_{xx} - \left(\frac{xt^2}{a_o^2N^2(\alpha)2!} + \frac{a_1x\,t^{\alpha+2}}{a_o^2N^2(\alpha)(\alpha+2)!} + \frac{a_1x\,t^{\alpha+1}}{a_o^2N^2(\alpha)(\alpha+1)!} + \frac{{a_1}^2xt^{2\alpha+2}}{{a_o}^2N^2(\alpha)(2\alpha+2)}\right)\right\}\right] + \left[\frac{a_1}{a_oN(\alpha)s^{\alpha+1}}\mathcal{L}\left\{\left(\frac{xt^2}{a_o^2N^2(\alpha)2!} + \frac{a_1x\,t^{\alpha+2}}{a_o^2N^2(\alpha)(\alpha+2)!} + \frac{a_1x\,t^{\alpha+1}}{a_o^2N^2(\alpha)(\alpha+1)!} + \frac{{a_1}^2xt^{2\alpha+2}}{{a_o}^2N^2(\alpha)(2\alpha+2)}\right)_{xx} - \left(\frac{xt^2}{a_o^2N^2(\alpha)2!} + \frac{a_1x\,t^{\alpha+2}}{a_o^2N^2(\alpha)(\alpha+2)!} + \frac{a_1x\,t^{\alpha+1}}{a_o^2N^2(\alpha)(\alpha+1)!} + \frac{{a_1}^2xt^{2\alpha+2}}{{a_o}^2N^2(\alpha)(2\alpha+2)}\right)\right\}\right]\right]$$

$$u_3(x,t) = -\frac{xt^3}{a_o^3N^3(\alpha)3!} - \frac{2a_1x\,t^{\alpha+3}}{a_o^3N^3(\alpha)(\alpha+3)!} - \frac{a_1x\,t^{\alpha+2}}{a_o^3N^3(\alpha)(\alpha+2)!} - \frac{{a_1}^2xt^{2\alpha+3}}{{a_o}^3N^3(\alpha)(2\alpha+3)!} - \frac{2{a_1}^2xt^{2\alpha+2}}{{a_o}^3N^3(\alpha)(2\alpha+2)!} - \frac{{a_1}^3xt^{3\alpha+3}}{{a_o}^3N^3(\alpha)(3\alpha+3)!} \tag{17}$$

Now, put the value of $N(\propto) = 1, a_1 = \frac{\alpha}{1-\alpha}$ and $a_o = \frac{1}{1-\alpha}$ in the above equation

$$u_3(x,t) = -\frac{\left(\frac{\alpha}{1-\alpha}\right)^3 xt^{3\alpha+3}}{\left(\frac{1}{1-\alpha}\right)^3(1)^3(3\alpha+3)!} \tag{18}$$

Now, put $\alpha = 1$

$$u_3(x,t) = -\frac{xt^6}{6!} \tag{19}$$

Thus, we can write the solution of $u(x,t)$ as:

$$u(x,t) = u_o(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) \ldots\ldots\ldots\ldots \tag{20}$$

$$u(x,t) = x + e^{-x} - \frac{xt^2}{2!} + \frac{xt^4}{4!} - \frac{xt^6}{6!} \ldots\ldots\ldots\ldots\ldots\ldots \tag{21}$$

$$u(x,t) = e^{-x} + x\left[1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} \ldots\ldots\ldots\ldots\ldots\ldots\ldots.\right] \tag{22}$$

which converge very rapidly to exact solution

$$u(x,t) = e^{-x} + x\cos x \tag{23}$$

Example No.2 Consider the following fractional equation.

$$u_t = u_{xx} + (-1 + cosx - sin^2x).u \tag{24}$$

and subject to the constant initial conditions

$$u(x,0) = \frac{1}{10}e^{cosx-11} \tag{25}$$

On both sides of Eq. (3.1) we take Laplace transform

$$\mathcal{L}(u_t) = \mathcal{L}\{u_{xx} + (-1 + cosx - sin^2x).u\} \tag{26}$$

$$S\mathcal{L}u(x,s) - u(x,0) = \frac{1}{a_oN(\propto)}\frac{s^\alpha + a_1}{s^\alpha}\mathcal{L}\{u_{xx} + (-1 + cosx - sin^2x).u\} \tag{27}$$

$$\mathcal{L}u(x,s) = \frac{u(x,0)}{s} + \frac{1}{s}\left[\frac{1}{a_oN(\alpha)}\frac{s^\alpha + a_1}{s^\alpha}\mathcal{L}\{u_{xx} + (-1 + cosx - sin^2x).u\}\right] \tag{28}$$

Now put the Eq. (3.2) in the above Eq.

$$\mathcal{L}u(x,s) = \frac{e^{cosx-11}}{10s} + \frac{1}{s}\left[\frac{1}{a_oN(\alpha)}\frac{s^\alpha + a_1}{s^\alpha}\mathcal{L}\{u_{xx} + (-1 + cosx - sin^2x).u\}\right] \tag{29}$$

On both sides of the Eq. (3.6), by applying inverse Laplace transform we obtain

$$u(x,t) = \mathcal{L}^{-1}\left[\frac{e^{cosx-11}}{10s} + \frac{1}{s}\left[\frac{1}{a_oN(\alpha)}\frac{s^\alpha + a_1}{s^\alpha}\mathcal{L}\{u_{xx} + (-1 + cosx - sin^2x).u\}\right]\right] \tag{30}$$

$$u(x,t) = \mathcal{L}^{-1}\left[\frac{e^{cosx-11}}{10s} + \frac{1}{a_oN(\alpha)}\frac{s^\alpha + a_1}{s^{\alpha+1}}\mathcal{L}\{u_{xx} + (-1 + cosx - sin^2x).u\}\right] \tag{31}$$

$$u(x,t) = \mathcal{L}^{-1}\left[\frac{e^{cosx-11}}{10s} + \frac{1}{a_oN(\alpha)}\frac{s^\alpha}{s^{\alpha+1}}\mathcal{L}\{u_{xx} + (-1 + cosx - sin^2x).u\} + \frac{1}{a_oN(\alpha)}\frac{a_1}{s^{\alpha+1}}\mathcal{L}\{u_{xx} + (-1 + cosx - sin^2x).u\}\right] \tag{32}$$

$$u(x,t) = \frac{e^{cosx-11}}{10} + \mathcal{L}^{-1}\left[\frac{1}{a_oN(\alpha)s}\mathcal{L}\{u_{xx} + (-1 + cosx - sin^2x).u\} + \frac{1}{a_oN(\alpha)}\frac{a_1}{s^{\alpha+1}}\mathcal{L}\{u_{xx} + (-1 + cosx - sin^2x).u\}\right] \tag{33}$$

By applying Homotopy perturbation transform algorithm (HPTA) on Eq. (3.10), we have

$$u(x,t) = \sum_{n=0}^{\infty} p^n u_n(x,t)$$

$$\sum_{n=0}^{\infty} p^n u_n(x,t) = \frac{e^{cosx-11}}{10} + p\mathcal{L}^{-1}\left[\left[\frac{1}{a_o N(\alpha)s}\mathcal{L}\{(\sum_{n=0}^{\infty} p^n u_n(x,t)_{xx}) + (\sum_{n=0}^{\infty} p^n u_n(x,t)(-1 + cosx - sin^2)x)\}\right] + \left[\frac{a_1}{a_o N(\alpha)s^{\alpha+1}}\mathcal{L}\{(\sum_{n=0}^{\infty} p^n u_n(x,t)_{xx}) - (\sum_{n=0}^{\infty} p^n u_n(x,t)(-1 + cosx - sin^2x))\}\right]\right] \quad (34)$$

Simplifying the Eq. (3.11) and comparing the coefficients of identical power of p, we get

For P_o:

$$u_0(x,t) = \frac{e^{cos-11}}{10} \quad (35)$$

For P_1:

$$u_1(x,t) = \mathcal{L}^{-1}\left[\left[\frac{1}{a_o N(\alpha)s}\mathcal{L}\left\{\left(\frac{e^{cos-11}}{10}\right)_{xx} + \left(\frac{e^{cos-11}}{10}\right)(-1 + cosx - sin^2x)\right\}\right] + \left[\frac{a_1}{a_o N(\alpha)s^{\alpha+1}}\mathcal{L}\left\{\left(\frac{e^{cos-11}}{10}\right)_{xx} + \left(\frac{e^{cos-11}}{10}\right)(-1 + cosx - sin^2x)\right\}\right]\right] \quad (36)$$

$$u_1(x,t) = \mathcal{L}^{-1}\left[\left[\frac{1}{a_o N(\alpha)s}\mathcal{L}\left\{\frac{(sin^2x - cosx).e^{cosx-11}}{10} + \left(\frac{e^{cos-11}}{10}\right)(-1 + cosx - sin^2x)\right\}\right] + \left[\frac{a_1}{a_o N(\alpha)s^{\alpha+1}}\mathcal{L}\left\{\frac{(sin^2x - cosx).e^{cosx-11}}{10} + \left(\frac{e^{cos-11}}{10}\right)(-1 + cosx - sin^2x)\right\}\right]\right] \quad (37)$$

$$u_1(x,t) = \mathcal{L}^{-1}\left[\left[\frac{1}{a_o N(\alpha)s}\mathcal{L}\left(-\frac{e^{cosx-11}}{10}\right)\right] + \left[\frac{a_1}{a_o N(\alpha)s^{\alpha+1}}\mathcal{L}\left(-\frac{e^{cosx-11}}{10}\right)\right]\right] \quad (38)$$

$$u_1(x,t) = \mathcal{L}^{-1}\left[\frac{-e^{cosx-11}}{10a_o.N(\alpha).s^2} - \frac{a_1 e^{cosx-11}}{10a_o.N(\alpha).s^{\alpha+2}}\right] \quad (39)$$

$$u_1(x,t) = \left[\frac{-e^{cosx-11}.t}{10a_o.N(\alpha)} - \frac{a_1 e^{cosx-11}.t^{\alpha+1}}{10a_o.N(\alpha).(\alpha+1)!}\right] \quad (40)$$

Now, put the value of $a_1 = \frac{\alpha}{1-\alpha}$ and $a_o = \frac{1}{1-\alpha}$ in the above equation

$$u_1(x,t) = \left[\frac{-(1-\alpha).e^{cosx-11}.t}{10.N(\alpha)} - \frac{\alpha.e^{cosx-11}.t^{\alpha+1}}{10.N(\alpha).(\alpha+1)!}\right] \quad (41)$$

Now, put $\alpha = 1$ and $N(\propto) = 1$

$$u_1(x,t) = -\frac{e^{cosx-11}}{10} \times \frac{t^2}{2!} \quad (42)$$

For P_2:

$$u_2(x,t) = \mathcal{L}^{-1}\left[\left[\frac{1}{a_o N(\alpha)s}\mathcal{L}\left\{\left(\frac{-e^{cosx-11}.t}{10a_o.N(\alpha)} - \frac{a_1 e^{cosx-11}.t^{\alpha+1}}{10a_o.N(\alpha).(\alpha+1)!}\right)_{xx} + \left(\frac{-e^{cosx-11}.t}{10a_o.N(\alpha)} - \frac{a_1 e^{cosx-11}.t^{\alpha+1}}{10a_o.N(\alpha).(\alpha+1)!}\right)(-1+cosx-sin^2x)\right\}\right] + \left[\frac{a_1}{a_o N(\alpha)s^{\alpha+1}}\mathcal{L}\left\{\left(\frac{-e^{cosx-11}.t}{10a_o.N(\alpha)} - \frac{a_1 e^{cosx-11}.t^{\alpha+1}}{10a_o.N(\alpha).(\alpha+1)!}\right)_{xx} + \left(\frac{-e^{cosx-11}.t}{10a_o.N(\alpha)} - \frac{a_1 e^{cosx-11}.t^{\alpha+1}}{10a_o.N(\alpha).(\alpha+1)!}\right)(-1+cosx-sin^2x)\right\}\right]\right] \quad (43)$$

$$u_2(x,t) = \mathcal{L}^{-1}\left[\left[\frac{1}{a_o N(\alpha)s}\mathcal{L}\left\{\left(\frac{-e^{cosx-11}(sin^2x-cosx).t}{10a_o.N(\alpha)} - \frac{a_1 e^{cosx-11}(sin^2x-cosx).t^{\alpha+1}}{10a_o.N(\alpha).(\alpha+1)!}\right) + \left(\frac{-e^{cosx-11}.t}{10a_o.N(\alpha)} - \frac{a_1 e^{cosx-11}.t^{\alpha+1}}{10a_o.N(\alpha).(\alpha+1)!}\right)(-1+cosx-sin^2x)\right\}\right] + \left[\frac{a_1}{a_o N(\alpha)s^{\alpha+1}}\mathcal{L}\left\{\left(\frac{-e^{cosx-11}(sin^2x-cosx).t}{10a_o.N(\alpha)} - \frac{a_1 e^{cosx-11}(sin^2x-cosx).t^{\alpha+1}}{10a_o.N(\alpha).(\alpha+1)!}\right) + \left(\frac{-e^{cosx-11}.t}{10a_o.N(\alpha)} - \frac{a_1 e^{cosx-11}.t^{\alpha+1}}{10a_o.N(\alpha).(\alpha+1)!}\right)(-1+cosx-sin^2x)\right\}\right]\right] \quad (44)$$

$$u_2(x,t) = \mathcal{L}^{-1}\left[\left[\frac{1}{a_o N(\alpha)s}\left\{\left(\frac{-e^{cosx-11}(sin^2x-cosx)}{10a_o.N(\alpha).s^2} - \frac{a_1 e^{cosx-11}(sin^2x-cosx)}{10a_o.N(\alpha).s^{\propto+2}}\right) + \left(\frac{-e^{cosx-11}}{10a_o.N(\alpha).s^2} - \frac{a_1 e^{cosx-11}}{10a_o.N(\alpha).s^{\propto+2}}\right)(-1+cosx-sin^2x)\right\}\right] + \left[\frac{a_1}{a_o N(\alpha)s^{\alpha+1}}\left\{\left(\frac{-e^{cosx-11}(sin^2x-cosx)}{10a_o.N(\alpha).s^2} - \frac{a_1 e^{cosx-11}(sin^2x-cosx)}{10a_o.N(\alpha).s^{\propto+2}}\right) + \left(\frac{-e^{cosx-11}}{10a_o.N(\alpha).s^2} - \frac{a_1 e^{cosx-11}}{10a_o.N(\alpha).s^{\propto+2}}\right)(-1+cosx-sin^2x)\right\}\right]\right] \quad (45)$$

$$u_2(x,t) = \mathcal{L}^{-1}\left[\left[\left\{\left(\frac{-e^{cosx-11}(sin^2x-cosx)}{10\,a_o^2.N^2(\alpha).s^3} - \frac{a_1 e^{cosx-11}(sin^2x-cosx)}{10.a_o^2.N^2(\alpha).s^{\propto+3}}\right) + \left(\frac{-e^{cosx-11}}{10\,a_o^2.N^2(\alpha).s^3} - \frac{a_1 e^{cosx-11}}{10\,a_o^2.N^2(\alpha).s^{\propto+3}}\right)(-1+cosx-sin^2x)\right\}\right] + \left[\left\{\left(\frac{-a_1 e^{cosx-11}(sin^2x-cosx)}{10\,a_o^2.N^2(\alpha).s^{\alpha+3}} - \frac{a^2{}_1 e^{cosx-11}(sin^2x-cosx)}{10\,a_o^2.N^2(\alpha).s^{2\propto+3}}\right) + \left(\frac{-a_1 e^{cosx-11}}{10\,a_o^2.N^2(\alpha).s^{\alpha+3}} - \frac{a^2{}_1 e^{cosx-11}}{10\,a_o^2.N^2(\alpha).s^{2\propto+3}}\right)(-1+cosx-sin^2x)\right\}\right]\right] \quad (46)$$

$$u_2(x,t) = \left[\left[\left\{\left(\frac{-e^{cosx-11}(sin^2x-cosx).t^2}{10\,a_o^2.N^2(\alpha).2!} - \frac{a_1 e^{cosx-11}(sin^2x-cosx).t^{\alpha+2}}{10.a_o^2.N^2(\alpha).(\alpha+2)!}\right) + \left(\frac{-e^{cosx-11}.t^2}{10\,a_o^2.N^2(\alpha).2!} - \frac{a_1 e^{cosx-11}.t^{\alpha+2}}{10\,a_o^2.N^2(\alpha).(\alpha+2)!}\right)(-1+cosx-sin^2x)\right\}\right] + \left[\left\{\left(\frac{-a_1 e^{cosx-11}(sin^2x-cosx).t^{\alpha+2}}{10\,a_o^2.N^2(\alpha).(\alpha+2)!} - \frac{a^2{}_1 e^{cosx-11}(sin^2x-cosx).t^{2\alpha+2}}{10\,a_o^2.N^2(\alpha).(2\alpha+2)!}\right) + \left(\frac{-a_1 e^{cosx-11}.t^{\alpha+2}}{10\,a_o^2.N^2(\alpha).(\alpha+2)!} - \frac{a^2{}_1 e^{cosx-11}.t^{2\alpha+2}}{10\,a_o^2.N^2(\alpha).(2\alpha+2)!}\right)(-1+cosx-sin^2x)\right\}\right]\right] \quad (47)$$

Now, put the value of $a_1 = \frac{\alpha}{1-\alpha}$ and $a_o = \frac{1}{1-\alpha}$ in the above equation

$$u_2(x,t) = \frac{\alpha^2.e^{cosx-11}.t^{2\alpha+2}}{10.N^2(\alpha).(2\alpha+2)!} \quad (48)$$

Now, put $\alpha = 1$ and $N(\propto) = 1$

$$u_2(x,t) = \frac{e^{cosx-11}}{10} \times \frac{t^4}{4!} \quad (49)$$

Thus, we can write the solution of $u(x,t)$ as:

$$u(x,t) = u_o(x,t) + u_1(x,t) + u_2(x,t) + u_3(x,t) \ldots\ldots\ldots\ldots\ldots\ldots \quad (50)$$

$$u(x,t) = \frac{e^{cosx-11}}{10} - \frac{e^{cosx-11}}{10} \times \frac{t^2}{2!} + \frac{e^{cosx-11}}{10} \times \frac{t^4}{4!} - \frac{e^{cosx-11}}{10} \times \frac{t^6}{6!} \ldots\ldots\ldots \quad (51)$$

$$u(x,t) = \frac{e^{cosx-11}}{10}\left[1 - \frac{t^2}{2!} + \frac{t^4}{4!} - \frac{t^6}{6!} \ldots\ldots\ldots\ldots\ldots\ldots\ldots.\right] \quad (52)$$

which converge very rapidly to exact solution

$$u(x,t) = \frac{e^{cosx-11}}{10}. \cos x \quad (53)$$

The numerical comes about of $u(x,t)$ for the approximate solution (3.33) discover by applying HPTM, the precise arrangement and the supreme mistake $E_T\left|u_{ex} - u_{app}\right|$for different values of t and x are delineated.

Conclusion

In this study, HPTM is connected to the arrangement of time fragmentary convection diffusion by utilizing ABC-operator with magnitude and porous impacts. Two case issues are discussed here to approve and test the effectiveness of the proposed strategy. One may see that the obtained results are in amazing assertion with HPM and ADM. The major good thing about this strategy over HPM and ADM is that typically a capable and viable strategy in finding the expository arrangement. It also helps to find the approximate solution for the partial differential equations for the fractional order in place of Adomian's polynomials. It is worth mentioning that the method is capable of reducing the volume of the computational work as compared to the classical methods showing an improvement in the performance of the approach.

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