Bayesian Analysis of Covid-19 mortality rate with Inverse Pareto distribution

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Abstract-Among different lethal viruses, in recent times the covid-19 was a worldwide pandemic, that the human race of the modern era of 21st century faced and it shut down the whole world for an instance. It not only effects the human race but it effects all the aspects for human life economic and social.

About 577M of cases and more than 6.4M deaths had been registered during the pandemic up until now. In Pakistan, up until now there are 1.55M cases and more than 30487 deaths had been recorded. This is crucial to apprehend the shape and flows of a disease whilst it input in a community. Moreover, to get rid of maximum loss we ought to recognize the predicted numbers of infectious patients, deaths, and the risk factors related to it. To estimate the predicted number of deaths from coronavirus, we use the Inverse Pareto distribution below Bayesian paradigm. The posterior distributions are derived assuming the non-informative priors (Uniform and Jeffery). The Bayesian estimation is done with each symmetrical and asymmetrical loss functions i.e. (Squared error, Quadratic error, Precautionary error, and weighted error).

Keywords: Inverse Pareto distribution, Uniform and Jeffery priors, loss functions, real data analysis.

Objectives:

- The fundamental motive of this study is to check the overall performance of every estimator under the uniform and Jeffery priors by using the real data in addition to simulated records.

Results:

In both cases under a uniform and Jeffery priors, the quadratic error loss function leads to the better estimation of the expected number of deaths due to coronavirus.

I. INTRODUCTION:

Coronavirus or COVID-19 is a well-known disease in the world. These are a group of related viruses that infect the mammals and birds (Ali, & Alharbi, 2020)[1]. The coronaviruses infect the human beings and can cause respiratory tract infections. These infections range from mild to fatal (Zumla, & Niederman, 2020)[2] that can cause death. Coronavirus disease (COVID-19) is caused by the SARS-CoV-2 virus (Yonker, Shen, & Kinane, 2020)[3] that is an infectious disease. Many people infected with COVID-19 experience minor to moderate symptoms and they do not need any special medical treatment. However, some people can become seriously ill and require acute medical attention (Cascella, et al., 2022[4]; Ali, & Alharbi, 2020[1]; Zumla, & Niederman, 2020[2]).

Acute respiratory syndrome coronavirus-2 (SARS-CoV-2) is a novel, severe, acute respiratory syndrome coronavirus. It was first identified from three people with pneumonia connected to the cluster of acute respiratory illness cases in Wuhan, a city of China. All structural features of the novel SARS-CoV-2 virus particle occur in related coronaviruses in nature. The first known infections from SARS-CoV-2 were discovered in Wuhan, China, on 31 December 2019.. The original source of viral transmission to humans remains unclear, as does whether the virus became pathogenic before or after the spillover event (Yonker, Shen, & Kinane, 2020[3]).

The virus primarily spreads between people through close contact and via aerosols and respiratory droplets that are exhaled when talking, breathing, or otherwise exhaling, as well as those produced from coughs or sneezes (Dhand, & Li, 2020[5]). Human coronaviruses are capable of causing illnesses ranging from the common cold to more severe diseases such as Middle East respiratory syndrome (MERS, fatality rate ~34%). SARS-CoV-2 is the seventh known coronavirus to infect people, after 229E, NL63, OC43, HKU1, MERS-CoV, and the original SARS-CoV (Yonker, Shen, & Kinane, 2020[3]).

Another area to know about the coronavirus is about the surfaces on which it may survive. It is not certain how long the virus that causes COVID-19 survives on surfaces, but it seems likely to behave like other coronaviruses (World Health Organization, 2020[6]). A recent review of the survival of human coronaviruses on surfaces found large variability, ranging from 2 hours to 9 days (Cascella, et al., 2022[4]). The survival time depends on a number of factors, including the type of surface, temperature, relative humidity and specific strain of the virus. The "Three C's" are a useful way to think about this. The World Health Organization describe settings where transmission of the COVID-19 virus spreads (World Health Organization, 2020[6]) more easily are; Crowded places, close-contact settings, especially where people have conversations very near each other and confined and enclosed spaces with poor ventilation.

In statistics, we mostly concern with the two primary philosophical perspectives one is the frequentist approach and the other one is the Bayesian. The frequentist or classical approach was established by Professor R.A. Fisher in a series of essential publications published about 1930 where the parameters are treated as fixed quantities. However, the parameters cannot be considered as a fixed quantity during the life testing period in many real-world circumstances including multivariable failure models, competing risks, dynamic reliability and therefore, the Bayesian framework is introduced. Since then many researchers used the Bayesian models, for example [7] provides a comprehensive treatment of Bayesian survival analysis, [8]. providing a broad coverage of the diverse aspects of reliability,[9] illustrates the creation of Bayesian assurance test plans for system reliability, [10] proposed a new analysis approach based on Bayesian inference principles for evaluating whether a measured bioassay, dosimetry, environmental monitoring. Recently, Bayesian estimation approach has received great attention by most researchers among them are AlAboud [11] using progressive censored data and asymmetric loss functions, exponential probability distribution under Bayesian framework has been studied by Sarhan [12]. Canavos and Taokas [13] utilized Weibull distribution, Guure et al. [14] used an extension of Jeffrey's prior to investigate Bayesian estimation for two-parameter of Weibull distributions. Elfessi and Reineke [15] discussed that how to determine the classical estimators under different choices with in a Bayesian Paradigm, Asgharzadeh [16] discussed the Bayesian estimation for the record values. Under the entropy loss, Shawn Ni and Dongchu Sun [17] investigated the Bayes estimator of the Linear Time Series (LTS) model. They developed the Bayes estimator and demonstrated that it involves frequentist regressor expectation.

They used a Markov Chain Monte Carlo approach thatsimulates the posteriors of the LTS parameters in conjunction with frequentist regressor expectation. They used Bayesian estimates to test an LTS model for seasonal impacts in some macroeconomic variables in the United States. Eskandarzadeh et al. explored the Bayesian estimation of the scale parameter of the exponential distribution under maximum ranked sampling. Singh and Kumar [18] defined the Bayesian estimation under a multiply type 2 censoring scheme. Ahmed et al. [9] considered Bayesian Survival Estimator for the Weibull distribution with censored data. Feroze [19] used multiple priors and loss functions to do Bayesian analysis of the scale parameter of an inverse Gaussian distribution. Using the squared error loss function, Almutairi and Heng [20] derived the shape parameter of the Generalized Power Distribution (GPD) using a Bayesian technique under noninformative (uniform) and informative (gamma) priors. Hasan and Baizid [21] discussed the Bayesian analysis of the parameter of Exponential distribution by using the Gamma prior with different loss. Sankudey [22] described the Bayesian Estimation of the Shape Parameter of the Generalized Exponential Distribution, Ijaz et al presented the Bayesian estimators of the parameter for the exponential distribution under Jeffery prior with various loss functions, a similar work on Bayesian analysis of the Shape Parameter of Lomax Distribution under various Priors with different loss functions has been studied by Ijaz et al [23]. Azam and Ahmad [24] using a Bayesian technique, estimated the scale parameter of the Nakagami distribution. Naji et al [25] discussed the Bayesian analysis for both Parameters of Gamma Distribution with precautionary loss function.

II. LIKELIHOOD FUNCTION

A random variable X follows Inverse Pareto distribution, by Guo, L., & Gui, W. (2018) [26] with parameters $\alpha \geq 0$ with the following

$$F(x)=\left(\frac{x}{1+x}\right)^{\alpha}, \alpha \geq 0, x \geq 0 \qquad 2.1$$

The corresponding probability density function is given by

$$f(x)=\frac{\alpha x^{\alpha-1}}{(1+x)^{\alpha+1}}, \alpha \geq 0, x \geq 0 \qquad 2.2$$

The log-likelihood function becomes

$$\prod_{i=0}^{n} f(x)=\alpha^{n}\left(\prod_{i=0}^{n}\frac{1}{x}\right)e^{-\alpha\sum_{i=0}^{n}\log\left(1+\frac{1}{x}\right)} \qquad 2.3$$

Bayesian Estimation of Inverse Pareto Distribution Posterior under Uniform Prior

The posterior probability distribution of the Inverse Pareto distribution under a uniform prior is characterized as

$$f(\alpha/x_i)=\frac{\prod_{i=0}^{n} f(x_i/\alpha)g(\alpha)}{\int_0^\infty \prod_{i=0}^{n} f(x_i/\alpha)g(\alpha)dx}$$

where $g(\alpha)\ \alpha\ \alpha$ is the prior probability function and $\prod_{i=0}^{n} f(x_i/\alpha)$ is the MLE of the Inverse Pareto distribution. Using Eq(2.3), we get

$$f(\alpha/x_i)=\frac{C^{(n+1)}}{\Gamma(n+1)}\alpha^n e^{-\alpha C} \qquad 2.4$$

This indicates that $f(\alpha/x_i)$, $Gamma(n+1, C)$, and $C=\sum_{i=0}^{n}\log\left(1+\frac{1}{x_i}\right)$

Posterior distribution under Jeffery Prior

The posterior probability distribution of the Inverse Pareto distribution under a uniform prior is characterized as

$$f(\alpha/x_i)=\frac{\prod_{i=0}^{n} f(x_i/\alpha)g(\alpha)}{\int_0^\infty \prod_{i=0}^{n} f(x_i/\alpha)g(\alpha)dx}$$

where $g(\alpha)=-E\left[\frac{\partial^2 \log L(x_i,\alpha)}{\partial\alpha^2}\right]$ is the prior probability function and $\prod_{i=0}^{n} f(x_i/\alpha)$ is the MLE of the Inverse Pareto distribution.Using Eq(2.3) , we get

$$f(\alpha/x_i)=\frac{C^{(n)}}{\Gamma(n)}\alpha^{n-1} e^{-\alpha C} \qquad 2.5$$

This indicates that $f(\alpha/x_i)$, $Gamma(n+1, C)$, and $C=\sum_{i=0}^{n}\left(1+\frac{1}{x}\right)$

Bayes Estimates under a Uniform Prior with different loss functions

This section explains the computation of Bayes estimators under a uniform prior with various loss functions.

Squared Error loss function (SELF)

The expression for SELF for α is defined by

$$\hat{\alpha}_{SELF}=(\hat{\alpha}-\alpha)^2$$

Utilizing the above expression for, we get

$$\hat{\alpha}\int\frac{C^{(n+1)}}{\Gamma(n+1)}\alpha^n e^{-\alpha C}d\alpha-\int\alpha\frac{C^{(n+1)}}{\Gamma(n+1)}\alpha^n e^{-\alpha C}d\alpha=0$$

The final result for α is given below

$$\hat{\alpha}_{SELF}=\frac{n+1}{C} \qquad 2.6$$

Quadratic Error loss function (QELF)

The QELF can be explained as

$$\hat{\alpha}_{QELF}=\left(\frac{\hat{\alpha}-\alpha}{\hat{\alpha}}\right)^2$$

Simplifying the above equation for α

$$\hat{\alpha}\int\frac{1}{\alpha}\frac{C^{(n+1)}}{\Gamma(n+1)}\alpha^n e^{-\alpha C}d\alpha-\alpha\int\frac{1}{\alpha^2}\frac{C^{(n+1)}}{\Gamma(n+1)}\alpha^n e^{-\alpha C}d\alpha=0$$

Simplifying the above expression, we get

$$\hat{\alpha}_{QELF} = \frac{n-1}{C} \qquad 2.7$$

Weighted Error loss function (WELF)

The WELF can be explained as below

$$\hat{\alpha}_{WELF} = \frac{(\hat{\alpha}-\alpha)^2}{\hat{\alpha}}$$

Simplifying the above equation for α

$$\hat{\alpha}\int \frac{1}{\alpha}\frac{C^{(n+1)}}{\overline{)(n+1)}}\alpha^n e^{-\alpha C} d\alpha - \int \frac{C^{(n+1)}}{\overline{)(n+1)}}\alpha^n e^{-\alpha C} d\alpha = 0$$

Simplifying the above expression, we get

$$\hat{\alpha}_{WELF} = \frac{n}{C} \qquad 2.8$$

Precautionary Error loss function (PELF)

The PELF can be defined as below

$$\hat{\alpha}_{PELF} = \frac{(\hat{\alpha}-\alpha)^2}{\hat{\alpha}}$$

Simplifying further the above equation for α

$$\int \frac{C^{(n+1)}}{\overline{)(n+1)}}\alpha^n e^{-\alpha C} d\alpha - \frac{1}{\hat{\alpha}^2}\int \alpha^2 \frac{C^{(n+1)}}{\overline{)(n+1)}}\alpha^n e^{-\alpha C} d\alpha = 0$$

Hence, we obtained the following expression

$$\hat{\alpha}_{PELF} = \frac{\sqrt{(n+1)(n+2)}}{C} \qquad 2.9$$

Bayes Estimates under Jeffery Prior with different loss functions

This section elaborates the derivation of Bayes estimates for various error loss functions under Jeffery prior

Squared Error loss function (SELF)

The expression for SELF for $\hat{\alpha}$ is defined by

$$\hat{\alpha}\int \frac{C^n}{\overline{)n}}\alpha^{n-1} e^{-\alpha C} d\alpha - \int \alpha \frac{C^n}{\overline{)n}}\alpha^{n-1} e^{-\alpha C} d\alpha = 0$$

In the end, we obtained the following result

$$\hat{\alpha}_{SELF} = \frac{n}{C} \qquad 2.10$$

Quadratic Error loss function (QELF)

The QELF can be explained as

The $\hat{\alpha}$ estimator can be explained by simplifying the following expression

$$\hat{\alpha}\int \frac{1}{\alpha^2}\frac{C^n}{\overline{)n}}\alpha^{n-1} e^{-\alpha C} d\alpha - \int \frac{1}{\alpha}\frac{C^n}{\overline{)n}}\alpha^{n-1} e^{-\alpha C} d\alpha = 0$$

After simplifying, we obtain the result below

$$\hat{\alpha}_{QELF} = \frac{n-2}{C} \qquad 2.11$$

Weighted Error loss function (WELF)

The WELF can be explained as below

$$\hat{\alpha}_{WELF} = \frac{(\hat{\alpha}-\alpha)^2}{\alpha}$$

Simplifying the above equation for α

$$\hat{\alpha}\int \frac{1}{\alpha}\frac{C^n}{\overline{)n}}\alpha^{n-1}e^{-\alpha C}d\alpha - \int \frac{C^n}{\overline{)n}}\alpha^{n-1}e^{-\alpha C}d\alpha = 0$$

Lastly, we obtained the following result

$$\hat{\alpha}_{WELF} = \frac{n-1}{C} \qquad 2.12$$

Precautionary Error loss function (PELF)

The PELF can be defined as below

$$\hat{\alpha}_{PELF} = \frac{(\hat{\alpha}-\alpha)^2}{\hat{\alpha}}$$

The Bayes estimator λ̂ PELF can be defined as

$$\int \frac{C^n}{\overline{)n}}\alpha^{n-1}e^{-\alpha C}d\alpha - \frac{1}{\hat{\alpha}^2}\int \alpha^2 \frac{C^n}{\overline{)n}}\alpha^{n-1}e^{-\alpha C}d\alpha = 0$$

Hence, we determined the result

$$\hat{\alpha}_{PELF} = \frac{\sqrt{n(n+1)}}{C} \qquad 2.13$$

Quantile Function

For a simulation study, we have considered the following quantile function

$$x = \frac{1}{U^{-1/\alpha} - 1}$$

where, U is distributed uniformly over the interval [0,1].

Simulation Study

In this study, we constructed a sample of size n for each estimator using the
Monte Carlo simulation method in the situation of uniform and Jeffery priors with a replication of w=5000. We have computed the estimated values of $\hat{\alpha}_{SELF}$, $\hat{\alpha}_{QELF}$, $\hat{\alpha}_{WELF}$ and $\hat{\alpha}_{PELF}$

A brief Monte Carlo simulation procedure using the Inverse Pareto distribution is given below;

1. Using the quantile function, generate random samples from the New distribution.
2. Calculate the Bayes estimator for numerous loss functions using the Jeffery and uniform priors.
3. The above procedure is repeated w times for each sample size and the average numbers of Bayes estimators and also MSE's are calculated.

Criteria to Decide the Best Estimator

To evaluate the performance and make a comparison among different estimators ($\hat{\alpha}_{SELF}$, $\hat{\alpha}_{QELF}$, $\hat{\alpha}_{WELF}$, $\hat{\alpha}_{PELF}$) the mean squared error (MSE) will be quantified with the following mathematical formula

$$MSE(\hat{\alpha}) = E(\hat{\alpha}-\alpha)^2 = \mathrm{var}(\hat{\alpha}) + \left[Bias(\hat{\alpha})\right]^2$$

Generally, the estimator with a smaller value of MSE will be considered as the best one among other estimators.

APPLICATION

This section elaborates a real data set of Covid-19 mortality rate which is recently cited by Farooq et.al [31] with the following information's

0.009, 0.014, 0.014, 0.023, 0.027, 0.032, 0.036, 0.041, 0.05, 0.054, 0.063, 0.095, 0.118, 0.122, 0.154, 0.181, 0.186, 0.213, 0.24, 0.258, 0.276, 0.294, 0.299, 0.389, 0.412, 0.421, 0.435, 0.503, 0.579, 0.611, 0.647, 0.761, 0.797, 0.91, 0.96, 1.073, 1.145, 1.218, 1.272, 1.322, 1.412, 1.553,1.743, 1.888, 1.992, 2.069, 2.155, 2.327, 2.553, 2.648, 2.712, 2.879, 2.983, 3.196, 3.336, 3.445, 3.486, 3.776, 3.776, 3.952, 4.088, 4.251, 4.459, 4.604, 4.83, 4.984, 5.129, 5.283, 5.419, 5.546, 5.704, 5.962, 6.315, 6.714, 6.985, 7.338, 7.642, 8.013, 8.321, 8.76, 9.063, 9.358, 9.833, 10.209, 0.666, 11.15, 11.15, 11.549, 12.354, 12.852, 13.468, 14.002, 14.618, 15.311, 15.849, 16.252, 16.728, 16.999, 17.669, 17.936, 18.267, 18.643, 18.864, 19.485, 19.897, 20.25, 20.603, 20.603, 20.911, 21.558, 21.907, 22.282, 22.559, 22.898, 23.192, 23.527, 23.84, 24.084, 24.383, 24.564, 24.564, 24.999, 25.207, 25.347, 25.528, 25.7, 25.845, 26.09, 26.198, 26.357, 26.357, 26.447, 26.551, 26.674, 26.818, 26.941, 26.941, 27.054, 27.158, 27.158, 27.226, 27.321, 27.398, 27.47, 27.534, 27.602, 27.67, 27.747, 27.792, 27.855, 27.896, 27.955, 27.955, 28.023, 28.073, 28.109, 28.154, 28.208, 28.267, 28.267, 28.317, 28.371, 28.403, 28.444, 28.448, 28.466, 28.494, 28.512, 28.647, 28.679, 28.702, 28.702, 28.724, 28.747, 28.788, 28.815, 28.838, 28.851, 28.878, 28.896, 28.924, 28.942, 28.969, 29.01, 29.041, 29.046, 29.064, 29.082, 29.118, 29.141, 29.173, 29.204, 29.231, 29.272, 29.308, 29.331, 29.354, 29.422, 29.458, 29.485, 29.485, 29.53, 29.585, 29.625, 29.662 ,29.689, 29.743, 29.788, 29.824, 29.883, 29.942, 29.974, 30.051, 30.123, 30.146, 30.209, 30.295, 30.341, 30.399, 30.454, 30.494, 30.508, 30.535, 30.599, 30.671, 30.762, 30.811, 30.888, 30.943, 31.006,31.088, 31.205, 31.341, 31.432, 31.545, 31.586, 31.69, 31.785, 31.939, 32.106, 32.183, 32.328, 32.414, 32.563, 32.731, 32.812, 34.229, 34.419, 34.687, 34.841, 35.058, 35.325, 35.506, 35.75, 35.954, 36.149, 36.33, 36.629, 36.968, 37.145, 37.394, 37.588, 37.851, 38.019, 38.421, 38.693, 38.947, 39.173, 39.494, 39.82, 39.983, 40.314, 40.789, 41.106, 41.486, 41.876, 42.238, 42.518, 42.89, 43.265, 43.768, 44.153, 44.438, 44.701, 44.95, 45.235, 45.484, 45.746, 46.068, 46.439, 46.679, 46.855, 47.123, 47.358.

Fig1 describe the kernel density, boxplot, TTT plot, and Q-Q plot of the data set.

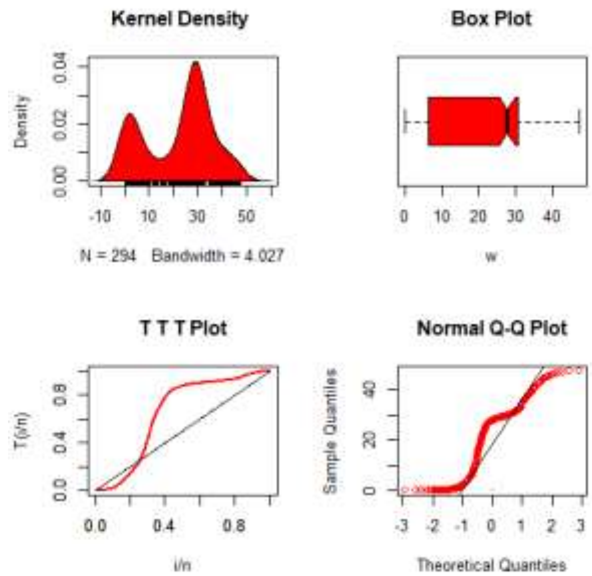

Fig1. Plots for the Covid-19 mortality rates

Table 1. Estimated value and MSE of α under uniform Prior, when n=80

∝	Criteria	BSE	BQEL	BWEL	BPEL
0.5	Estimated value	3.3068	3.2417	3.2768	3.3624
	MSE	11.3123	10.8845	11.1236	11.7324
1.5	Estimated value	3.3447	3.2659	3.2976	3.36009
	MSE	11.6601	11.0887	11.2874	11.7378
2.5	Estimated value	3.3354	3.2397	3.2859	3.3561
	MSE	11.5603	10.8889	11.2091	11.693
3.5	Estimated value	3.3295	3.2468	3.2807	3.3764
	MSE	11.5019	10.9343	11.1311	11.8478
4.5	Estimated value	3.3383	3.209	3.232	3.3278
	MSE	11.5406	10.4601	10.6011	11.4761

Table 1 represents the estimated values and MSE's for various values of $\hat{\alpha}$ at fixed samples and other parameter values. The table shows that the MSE for the Bayes estimator of Quadratic Error Loss function as compared to other loss functions in smaller, but much closer to the MSE of the Weighted Error Loss function.

Table 2. Estimated value and MSE of α under uniform Prior, when α =2.5

n	Criteria	BSE	BQEL	BWEL	BPEL
20	Estimated value	4.6622	4.1835	4.3497	4.6802
	MSE	34.656	27.774	29.3034	33.8807
40	Estimated value	3.7021	3.5184	3.5951	3.70423
	MSE	16.4247	14.6777	15.28	16.1857
60	Estimated value	3.42485	3.3181	3.3775	3.4843
	MSE	12.6455	11.8318	12.2196	13.0736
80	Estimated value	3.34094	3.23224	3.304112	3.3596
	MSE	11.5541	10.8096	11.3208	11.7566
90	Estimated value	3.26	3.2	3.23	3.28
	MSE	10.81	10.4	10.6	10.9

Table 2 represents the estimated values and MSE's for various values of different samples at $\hat{\alpha}$ fixed values and other parameter values. The table shows that the MSE for the Bayes estimator of Quadratic Error Loss function as compared to other loss functions in smaller, but much closer to the MSE of the Weighted Error Loss function.

Fig 2 clearly shows that BQEL performs well and become closer to BWEL when the sample size is large. Similarly, the behavior of BSE and BPEL much similar when the sample size is increases.

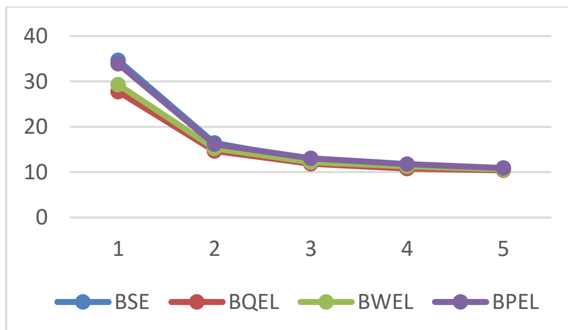

Figure 2. Graph of MSE for of α under a uniform Prior, when α =2.5

Table 3. Estimated value and MSE of α under Jeffery Prior, when n=80

∝	Criteria	BSE	BQEL	BWEL	BPEL
1.5	Estimated value	3.2790	3.2050	3.2049	3.2893
	MSE	11.1248	10.6415	10.4351	11.2269
2.5	Estimated value	3.2722	3.2160	3.2459	3.3102
	MSE	11.0874	10.7053	10.8926	11.3301
3.5	Estimated value	3.2939	3.2144	3.2546	3.2982

	MSE	11.2603	10.7208	10.9721	11.2798
4.5	Estimated value	3.2904	3.2019	3.2699	3.3130
	MSE	11.2186	10.5897	11.1113	11.3849
5.5	Estimated value	3.2977	3.2161	3.2262	3.3018
	MSE	11.2683	10.7128	10.6868	11.3097

Table 3 represents the estimated values and MSE's for various values of $\hat{\alpha}$ at fixed samples and other parameter values. The table shows that the MSE for the Bayes estimator of Quadratic Error Loss function as compared to other loss functions in smaller, but much closer to the MSE of the Weighted Error Loss function.

Table 4. Estimated value and MSE of α under Jeffery Prior, when α =1.5

n	Criteria	BSE	BQEL	BWEL	BPEL
20	Estimated value	4.3928	4.0236	4.2686	4.5130
	MSE	30.3261	25.4833	29.0821	31.6074
40	Estimated value	3.6314	3.4008	3.4998	3.6559
	MSE	15.5897	13.4976	14.4444	15.9376
60	Estimated value	3.3738	3.2735	3.3328	3.4261
	MSE	12.2626	11.5768	12.0171	12.7130
80	Estimated value	3.2910	3.2126	3.2515	3.3246
	MSE	11.2701	10.7167	10.9972	11.4606
90	Estimated value	3.2500	3.1900	3.2300	3.2900
	MSE	10.8000	10.4000	10.7000	11.1000

Table 4 represents the estimated values and MSE's for various values of different samples at fixed values $\hat{\alpha}$ and other parameter values. The table shows that the MSE for the Bayes estimator of Quadratic Error Loss function as compared to other loss functions in smaller, but much closer to the MSE of the Weighted Error Loss function.

Fig 3 also demonstrate that BQEL perform well as compared to others but as the sample size is increases, BQEL, BWEL, and BPEL are converges rapidly to each other's.

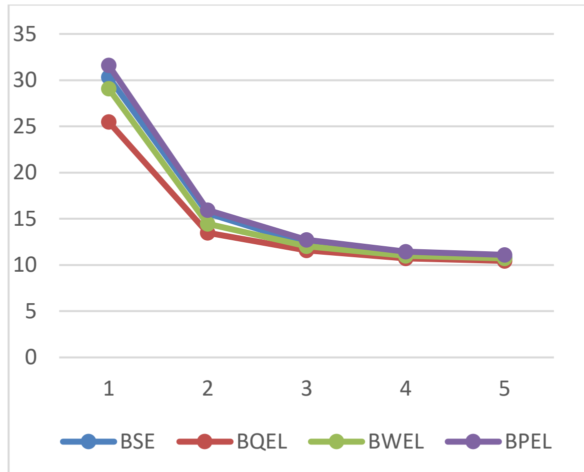

Fig 3. Graph of MSE for of α under a Jeffery Prior, when α =1.5

Simulation study

In this section, a simulation study is conducted by choosing different values of parameters with replication of 5000. Following are the numerical results

Table 5. Estimated value and MSE of α under uniform Prior, when n=80

∝	Criteria	BSE	BQEL	BWEL	BPEL
0.5	Estimated value	0.5127	0.5005	0.5053	0.516
	MSE	0.1993	0.1886	0.1928	0.2022
1.5	Estimated value	1.5428	1.5001	1.5166	1.5468
	MSE	2.199	2.0744	2.1227	2.2113
2.5	Estimated value	2.5637	2.5047	2.5311	2.5798
	MSE	6.3054	6.009	6.1367	6.3883
3.5	Estimated value	3.5826	3.5081	3.5401	3.6086

	Criteria	BSE	BQEL	BWEL	BPEL
	MSE	12.5003	11.9841	12.2046	12.6903
4.5	Estimated value	4.6226	4.4979	4.5502	4.6328
	MSE	21.0138	19.8632	20.3378	21.0917

Table 5 represents the estimated values and MSE's for various values of $\hat{\alpha}$ at fixed samples and other parameter values. The table shows that the MSE for the Bayes estimator of Quadratic Error Loss function as compared to other loss functions in smaller, but much closer to the MSE of the Weighted Error Loss function.

Table 6. Estimated value and MSE of α under uniform Prior, when α =2.5

n	Criteria	BSE	BQEL	BWEL	BPEL
10	Estimated value	3.0523	2.5072	2.7937	3.1723
	MSE	10.0878	6.7922	8.3749	10.8749
20	Estimated value	2.7331	2.5014	2.6372	2.8267
	MSE	7.6204	6.2603	6.9598	8.052
30	Estimated value	2.6754	2.5024	2.583	2.7267
	MSE	7.0474	6.141	6.5644	7.321
40	Estimated value	2.6199	2.5062	2.5648	2.6593
	MSE	6.6825	6.1073	6.3967	6.8908
50	Estimated value	2.6016	2.4944	2.5434	2.6252
	MSE	6.544	6.0055	6.253	6.6723

Table 6 represents the estimated values and MSE's for various values of different samples at fixed values $\hat{\alpha}$ and other parameter values. The table shows that the MSE for the Bayes estimator of Quadratic Error Loss function as compared to other loss functions in smaller, but much closer to the MSE of the Weighted Error Loss function.

Figure 4 explore that the best results can be achieved with the BQEL when the estimator is deal with the uniform priors.

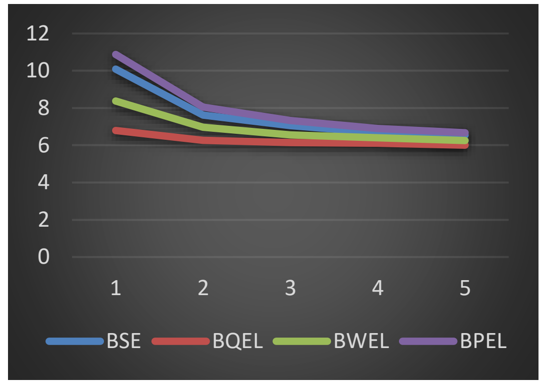

Figure 4. Graph of MSE for of α under a Jeffery Prior, when α =2.5

Table 7. Estimated value and MSE of α under uniform Prior, when n=80

∝	Criteria	BSE	BQEL	BWEL	BPEL
1.5	Estimated value	1.5197	1.4813	1.4984	1.5317
	MSE	2.1323	2.0202	2.0691	2.1678
2.5	Estimated value	2.5351	2.4687	2.4985	2.5461
	MSE	6.1631	5.8320	5.9771	6.2157
3.5	Estimated value	3.5409	3.4623	3.5020	3.5684
	MSE	12.2111	11.6564	11.9358	12.4071
4.5	Estimated value	4.5548	4.4494	4.4913	4.5757
	MSE	20.3761	19.4250	19.8042	20.5711
5.5	Estimated value	5.5821	5.4228	5.5000	5.5971
	MSE	30.7811	29.0328	29.8800	30.9464

Table 7 represents the estimated values and MSE's for various values of α at fixed samples and other parameter values. The table shows that the MSE for the Bayes estimator of Quadratic Error Loss function as compared to other loss functions in smaller, but much closer to the MSE of the Weighted Error Loss function.

Table 8. Estimated value and MSE of α under uniform Prior, when α =1.5

n	Criteria	BSE	BQEL	BWEL	BPEL
10	Estimated value	2.7608	2.2146	2.4972	2.9285
	MSE	8.16	5.1902	6.6208	9.224
20	Estimated value	2.6379	2.3697	2.5162	2.6891
	MSE	6.9779	5.5929	6.3551	7.2723
30	Estimated value	2.5927	2.4077	2.5156	2.6308
	MSE	6.61	5.6738	6.2064	6.8006
40	Estimated value	2.5584	2.4322	2.4899	2.5877
	MSE	6.3631	5.7337	6.0175	6.5178
50	Estimated value	2.5487	2.4526	2.4964	2.5736
	MSE	6.2787	5.8042	6.0167	6.4043

Table 8 represents the estimated values and MSE's for various values of different samples at fixed values $\hat{\alpha}$ and other parameter values. The table shows that the MSE for the Bayes estimator of Quadratic Error Loss function as compared to other loss functions in smaller, but much closer to the MSE of the Weighted Error Loss function.

Figure 5 declared that under the Jeffery prior, the better performance can be achieved with the BQEL estimator.

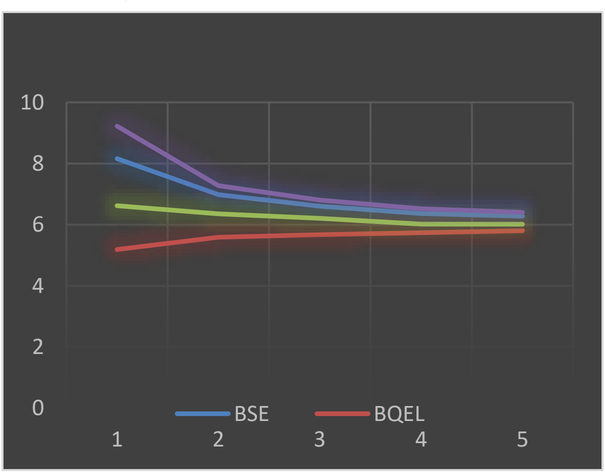

Fig 5. Graph of MSE for of α under a Jeffery Prior, when α =1.5

Conclusion

Bayesian analysis remained the key approach for modeling different type of diseases because of having the prior information. The purpose of such type of modeling is to determine the expected number of infectious patients and deaths from it. This study focuses on the Bayesian analysis of the Inverse Pareto distribution with non-informative (Uniform and Jeffery) priors under four loss functions. The mathematical work is supported by using the real data of Covid-19 mortality rates in Pakistan. The results show that considering a uniform prior, Bayes estimator of the quadratic error loss function (BQEL) perform well than the other loss functions. Moreover, it's clearly showing that as the sample size increases, the MSE of the shape parameter of the Inverse Pareto distribution in BSE and BPEL loss function turn out to be equal as portrayed in Figure 2. In the case of Jeffery prior, the quadratic error loss function (BQEL) provides better results as compared to other approaches. Figure 3 also illustrates that as the sample size increases, the BSE approach and the BPEL hastily become indistinguishable. We conclude that the Quadratic error loss function under both priors lead to the better estimation of the mortality rate of the Covid-19 in Pakistan.

References:

[1] Imran Ali and Omar ML Alharbi. Covid-19: Disease, management, treatment, and social impact. *Science of the total Environment*, 728:138861, 2020.

[2] Alimuddin Zumla and Michael S Niederman. The explosive epidemic outbreak of novel coronavirus disease 2019 (covid-19) and the persistent threat of respiratory tract infectious diseases to global health security. *Current opinion in pulmonary medicine*, 2020.

[3] Lael M Yonker, Kunling Shen, and T Bernard Kinane. Lessons unfolding from pediatric cases of covid-19 disease caused by sars-cov-2 infection. *Pediatric pulmonology*, 55(5):1085, 2020.

[4] Cascella, M., Rajnik, M., Aleem, A., Dulebohn, S. C., & Di Napoli, R. (2022). Features, evaluation, and treatment of coronavirus (COVID-19). *Statpearls [internet]*.

[5] Dhand, R., & Li, J. (2020). Coughs and sneezes: their role in transmission of respiratory viral infections, including SARS-CoV-2. *American journal of respiratory and critical care medicine*, *202*(5), 651-659.

[6] World Health Organization. (2020). *Water, sanitation, hygiene and waste management for COVID-19: technical brief, 03 March 2020* (No. WHO/2019-NcOV/IPC_WASH/2020.1). World Health Organization.

[7]] Joseph G Ibrahim, Ming-Hui Chen, Debajyoti Sinha, JG Ibrahim, and MH Chen. *Bayesian survival analysis*, volume 2. Springer, 2001.

[8] MS Hamada, AG Wilson, BP Weaver, RW Griffiths, and HF Martz. Bayesian binomial assurance tests for system reliability using component data. *Journal of Quality Technology*, 46(1):24–32, 2014.

[9] A.O.M Ahmed, NA Ibrahim, J Arasan, and MB Adam. Extension of jeffreys' prior estimate for weibull censored data using lindley's approximation. *Australian Journal of Basic and Applied Sciences*, 5(12):884–889, 2011.

[10] G Miller, WC Inkret, and HF Martz. Bayesian detection analysis for radiation exposure. *Radiation Protection Dosimetry*, 48(3):251–256, 1993.

[11] Fahad M Al-Aboud. Bayesian estimations for the extreme value distribution using progressive censored data and asymmetric loss. In *International Mathematical Forum*, volume 4, pages 1603–1622. Citeseer, 2009.

[12] A Sarhan and A Karawia. Empirical bayes estimators for the exponential reliability measures. *INTERNATIONAL JOURNAL OF APPLIEDMATHEMATICS*, 1(5):521–528, 1999.

[13] George C Canavos and Chris P Taokas. Bayesian estimation of life parameters in the weibull distribution. *Operations Research*, 21(3):755–763, 1973.

[14] Chris Bambey Guure, Noor Akma Ibrahim, and Al Omari Mohammed Ahmed. Bayesian estimation of two-parameter weibull distribution using extension of jeffreys' prior information with three loss functions. *Mathematical Problems in Engineering*, 2012, 2012.

[15]] Abdulaziz Elfessi and David M Reineke. A bayesian look at classical estimation: the exponential distribution. *Journal of Statistics Education*, 9(1),2001.

[16] A Asgharzadeh. On bayesian estimation from exponential distribution based on records. *Journal of the Korean Statistical Society*, 38(2):125–130, 2009.

[17] Shawn Ni and Dongchu Sun. Intrinsic bayesian estimation of linear time series models. *Statistical Theory and Related Fields*, 5(4):275–287, 2021.

[18] Umesh Singh and Anil Kumar. Bayesian estimation of the exponential parameter under a

multiply type-ii censoring scheme. *Austrian Journal of Statistics*, 36(3):227–238, 2007.

[19] Navid Feroze. Estimation of scale parameter of inverse gausian distribution under a bayesian framework using different loss functions. *Scientific Journal of Review*, 1(3):39–52, 2012.

[20] AO Almutairi and CL Heng. Bayesian estimate for shape parameter from generalized power function distribution. *Mathematical Theory and Modelling*, 2:2224–5804, 2012.

[21] MR Hasan and AR Baizid. Bayesian estimation under different loss functions using gamma prior for the case of exponential distribution. *Journal of Scientific Research*, 9(1):67–78, 2017.

[22] Sanku Dey. Bayesian estimation of the shape parameter of the generalized exponential distribution under different loss functions. *Pakistan Journal of Statistics and Operation Research*, pages 163–174, 2010.

[23] IJAZ Muhammad. Bayesian estimation of the shape parameter of lomax distribution under uniform and jeffery prior with engineering applications. *Gazi University Journal of Science*, pages 1–1, 2021.

[24] Azam Zaka and Ahmad Saeed Akhter. Bayesian approach in estimation of scale parameter of nakagami distribution. *Pakistan Journal of Statistics and Operation Research*, pages 217–228, 2014.

[25] Loaiy F Naji and Huda A Rasheed. Bayesian estimation for two parameters of gamma distribution under generalized weighted loss function. *Iraqi Journal of Science*, 60(5):1161–1171, 2019.

[26] Guo, L., & Gui, W. . Bayesian and classical estimation of the inverse Pareto distribution and its application to strength-stress models. *American Journal of Mathematical and Management Sciences*, *37*(1), 80-92, 2018.

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